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BIG BANG VERSUS STEADY STATE
ARE THESE INCOMPATIBLE IDEAS?

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Summary

In the present paper it is shown how it is possible to use the strict "light principle" as a point of departure for deriving three new "steady state" models of the universe which are at variance with the Robertson Walker Metric but fulfil Milne's cosmological principle.

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Contents

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A. INTRODUCTION

The γ-factor, which is defined as the quotient between an element of frame time dt and an element of proper time $d\tau$, is the most noticeable consequence of special relativity ***(SR)***.

A standard clock passing along a series of slave clocks, distributed over the co-moving ("stationary") frame of an observer and synchronized in the conventional way to the master clock of that observer, will thus appear retarded according to: $\gamma = dt/d\tau = \sqrt{1-dr^2/dt^2}$.

On the other hand, $d\tau^2 = dt^2 - dr^2 = dt'^2 - dr'^2$ is usually seen as a direct consequence of the differential Lorentz Transformations ***(LT′)*** in agreement with the relativity principle ***(RP)*** and the principle of a constant light speed, here termed the "light principle" ***(LP)***.

How can $d\tau$ be delayed relative to dt and yet be invariant? In order to solve this problem we must pass on to cosmology. One of the first "big bang" models, probably the only one based on the integral Lorentz Transformations is the uniform expansion model of E.A. Milne [1935]. In a paper on Milne's theory of kinematic relativity ***(KR)*** his former student A.G. Walker [1937] showed how the model can be restated so as to accomodate a cosmic time, $\mathcal{T}$:

$$t = \tau\, ch\sigma \ \ . \ r = \tau\, sh\sigma$$
$$d\mathcal{T}^2 = dt^2 - dr^2 = d\tau^2 - \tau^2 d\sigma^2$$
$$d\sigma \to 0 \ . \Rightarrow . \ d\tau \to d\mathcal{T}$$

This result inspired him to devise a method for the development of models incompatible with ***LT***, and thus to develop his own version of the famous Robertson-Walker metric ***(RWM)***:

$$(1) \qquad d\mathcal{T}^2 = d\tau^2 - \mathcal{S}^2(\tau)\, d\sigma^2 = invar.$$

Here τ is a universal parameter, $\mathcal{S}(\tau)$ is a universal scale function and σ a fixed ("co-moving") coordinate characterizing one fundamental observer relative to another.

Probably the Milne model is unique in the manner that it is the only model to combine ***LP*** in the strict sense with ***RWM***. Thus, when Bondi & Gold presented their "steady state" ***(SS)*** model, they explicitly based it on ***RWM***, noticing that their model is incompatible with ***LT***.

The scale function $\mathcal{S}$ of their ***SS*** universe being $\mathcal{S}_1(\tau) \equiv e^\tau$, it is easy to demonstrate that the model of Bondi & Gold with standard definitions of t & r is incompatible with a strong ***LP***:

$$d\mathcal{T}^2 = d\tau^2 - e^{2\tau} d\sigma^2 = (dt^2 - dr^2)(1 - th^2 r)$$

This is shown in the appendix where two other models having scale functions $\mathcal{S}_2(\tau) \equiv sh\,\tau$ and $\mathcal{S}_3(\tau) \equiv ch\,\tau$, both asymptotic approximations to $\mathcal{S}_1(\tau) \equiv e^\tau$, are derived from ***RWM***.

But incompatibility of $\mathcal{S}_1(\tau)$ with ***LT*** does not entail incompatibility of $\mathcal{S}_1(\tau)$ with ***LT′***, the differential ***LT***. So we may ask if it is possible to devise a new ***SS*** model by means of ***LT′*** & ***LP***, using the method of Milne, instead of basing it on ***RWM***, using the method of Walker.

In this way we are faced with a choice between $d\mathcal{T}^2 = dt^2 - dr^2$ together with ***LT′***, and $d\mathcal{T}^2 = d\tau^2 - e^{2\tau} d\sigma^2$ in combination with some other and hitherto unexplored transformations. Following Bondi and choosing the standard solution, we have a cosmic time together with what Milne called a *public* 3-space. Making the other choice, our 3-space is *private* in the sense that it can only be described in the perspective of an observer. But we still have a cosmic time, $\mathcal{T}$.

Hence our problem: Is a cosmology based on ***LP*** in a strong sense at all feasible?

1. A SIMPLE DERIVATION OF LT

A simple way of obtaining the Lorentz Transformations ***(LT)*** of Special Relativity ***(SR)*** - not rigorous, but illuminating - would begin with the Galileo Transformations ***(GT)***, unprimed coordinates referring to an observer O and primed to another observer O', taking O & O' to be in collinear motion with uniform relative velocity v, as reckoned from O to O':

$$t' = t \ . \ x' = x - vt \ . \ y' = y \ . \ z' = z$$

GT conform to the relativity principle ***(RP)***, but not to that of a constant light speed ***(LP)***. Hence, if O coincide with O' at the event $(t_o, x_o, 0, 0) = (t'_o, x'_o, 0, 0) = (0, 0, 0, 0)$, a light wave emerging from that event could not be spherical with respect to both observers at the same time, as equation $c^2t^2 = x^2 + y^2 + z^2$ is not transformed into the similar equation $c^2t'^2 = x'^2 + y'^2 + z'^2$. So we cannot put $c \equiv c'$; this shows that neither can we synchronize clocks using reflected light, nor can we interpret spatial distance as light-time measured by reflected radar signals.

If, by contrast, we insist on $c \equiv c'$, in *apparent* agreement with the results of observation and experiment, we have to modify ***GT*** accordingly. However, we may preserve the standard convention $v' = -v$, now enlightened by the fact that the numerical value of both velocities are expressible as the same fraction of the speed of light: $|v|/c = |v'|/c$. A further simplification is obtained if we put $c \equiv c' \equiv 1$, thus interpreting all speeds as fractions of the speed of light. How should ***GT*** be modified? The only parameter we dispose of is $|v'| = |v| =$ *const*.

So let us define a new function $\gamma \equiv \gamma(v)$. According to ***RP***, γ would have to be invariant. The simplest assumption is that γ only affects transformations in the direction of the x-axis, leaving the other spatial dimensions unaffected. But t being involved in the transformation of x to x', it may affect t' too, thus necessitating a renunciation of the classical transformation $t = t'$. An additive constant would hardly do the job. Let us try if γ could be a multiplicative factor:

(2) $$x' = \gamma\,(x - vt) \ . \ y' = y \ . \ z' = z \ . \ x = \gamma\,(x' - v't')$$

Assuming $v' = -v$, it must be done this way, for v is the velocity of a fix-point in the frame of O' as calculated by O: $dx' = \gamma(dx - vdt) = 0 \ \Rightarrow \ dx = vdt$, just as v' is the velocity of a fix-point in the frame of O as calculated by O': $dx = \gamma(dx' - v'dt') = 0 \ \Rightarrow \ dx' = v'dt'$. From $x' = \gamma(x - vt)$ combined with $x = \gamma(x' + vt')$ we then derive the transformation for time:

$$x = \gamma(x' + vt') = \gamma\{\gamma(x - vt) + vt'\} \ \Rightarrow \ t' = \gamma\{t - x(1 - \gamma^{-2})/v\}$$

In case of light propagation, as seen above: $t^2 - (x^2 + y^2 + z^2) = t'^2 - (x'^2 + y'^2 + z'^2) = 0$. Further, all velocities being fractions of the velocity of light, the following must hold in general:

$$t^2 - (x^2 + y^2 + z^2) = t'^2 - (x'^2 + y'^2 + z'^2)$$

On account of $y' = y$ & $z' = z$ we accordingly obtain: $t^2 - x^2 = t'^2 = x'^2$. It is therefore evident that the temporal co-ordinates t & t' must transform in the same way as the spatial ones x & x':

(3) $$t' = \gamma\{t - x(1 - \gamma^{-2})/v\} = \gamma(t - vx)$$

From this it is easy to derive a precise expression for the γ-factor; we obtain:

(4) $$\gamma = 1/\sqrt{1 - v^2}$$

Hence we shall claim that eqs. $x' = \gamma(x - vt)$ & $x = \gamma(x' + vt')$ contain the very germ of ***LT***.

2. THE IMPORTANCE OF THE γ-FACTOR

Granted ***LP*** and $c \equiv c' \equiv 1$, the natural definition of coordinates is the standard one:

$$\tau_3 \equiv t + x \ \Downarrow \ \tau_1 \equiv t - x$$
$$t = \tfrac{1}{2}(\tau_3{+}\tau_1) \ . \ x = \tfrac{1}{2}(\tau_3{-}\tau_1)$$
$$\tau_4' \equiv t' + x' \ \Downarrow \ \tau_2' \equiv t' - x'$$
$$t' = \tfrac{1}{2}(\tau_4'{+}\tau_2') \ . \ x' = \tfrac{1}{2}(\tau_4'{-}\tau_2')$$

Why 'τ' ? The point is that τ_i & τ_i' are read directly off the master-clocks of O & O', resp., denoting the proper times of those clocks, whereas t & t' are read off the slave-clocks fixed to the co-moving frames F_o & $F_{o'}$ of O & O', resp., denoting the standard frame times of F_o & $F_{o'}$ resp. t & t'. So *master-clocks* show *proper times* τ while *slave clocks* show *frame times* t.

Imagine a zig-zag signal passing to & fro between O & O' directly, without any delay:

$$... < \tau_1 < \tau_2' < \tau_3 < \tau_4' < ...$$

These are epochs taken to be read off the master-clocks of O & O' at the events of reflection. Granted ***RP***, τ_4' must be the same function of τ_3 as τ_3 of τ_2', and as τ_2' of τ_1, viz. $\tau_{i+1} \equiv \psi(\tau_i)$:

(5) $$\tau_4' = \psi(\tau_3) \ . \ \tau_3 = \psi(\tau_2') \ . \ \tau_2' = \psi(\tau_1)$$

The signal-function ψ forms the core of the Milne-Whitrow derivation of ***LT***, hailed as *transcendental* by J.R. Lucas [1971]. Taking the relative velocities of O & O' to be uniform and reciprocal, $v \equiv -v'$, the function ψ must be linear, of the form $s\,\tau{+}k$. The Doppler Shift (DS), which we identify with its derivative: $d\psi/d\tau = s$, must then be both constant and reciprocal:

(6) $$1{+}z \equiv d\tau_3/d\tau_2' = d\tau_2'/d\tau_1 \equiv 1{+}z'$$

(7) $$1{+}z = \sqrt{d\tau_3/d\tau_1} = \sqrt{\tfrac{dt+dx}{dt-dx}} = \sqrt{\tfrac{1+v}{1-v}} = e^{arth\, v}$$

Synchronizing the master-clocks of O & O' to read $\tau = \tau' = 0$ by coincidence, we choose $k \equiv 0$ for signals exchanged directly between O & O'; hence $\tau_3/\tau_2' = s = \tau_2'/\tau_1$. Further:

$$\tau_2' = \sqrt{\tau_3\tau_1} = \sqrt{(t{+}x)(t{-}x)} = t\sqrt{1{-}x^2/t^2}$$
$$\tau_3 = \sqrt{\tau_4'\tau_2'} = \sqrt{(t'{+}x')(t'{-}x')} = t'\sqrt{1{-}x'^2/t'^2}$$

Thus, in case of "photon" transmission, $\frac{x}{t} = \frac{x'}{t'} = 1$, the element of proper time $d\tau$ will be zero even though the element of frame time dt is not. Further, for the relative motion of O & O':

$$x' = 0 \ \Rightarrow: \ x = v\,t \ \Rightarrow \ dt = d\tau'/\sqrt{1{-}v^2} = \gamma\, d\tau'$$
$$x = 0 \ \Rightarrow: \ x' = v't' \ \Rightarrow \ dt' = d\tau/\sqrt{1{-}v'^2} = \gamma\, d\tau$$

This proves that the master-clock of O' showing $d\tau'$ will appear to be retarded relative to a series of slave-clocks distributed on the x-axis of O, just as the master-clock of O showing $d\tau$ will appear to be retarded relative to a series of slave-clocks distributed along the x'-axis of O'.

So proper time differs from frame time, there is no surprise in this, cf. H. Arzeliés [1966]. What *is* surprising, however, is that in case of energy change, due to the performance of work, the retardation will be absolute. Thus a moving clock will lack behind a resting clock of the same construction if it returns to its point of departure after having completed a circuit in space.

Nevertheless, as far as inertial collinear motion is concerned, we can reduce ***LT*** to ***GT*** for any pair of observers! This will be demonstrated in the following section.

3. THE FORMAL REDUCTION OF LT TO GT

With $\gamma = 1/\sqrt{1-v^2}$, as shown, the standard ***LT*** can be stated in the following form:

(8) $$x' = \gamma(x - vt) \ . \ y' = y \ . \ z' = z \ . \ x = \gamma(x' - v't')$$

Insert the values of t & t' from $\tau \equiv t - \frac{x}{v}(1-\gamma^{-1}) \equiv t' - \frac{x'}{v'}(1-\gamma^{-1})$ into ***LT***, then, lo and behold: we immediately obtain something which is surprisingly similar to the ***GT*** of classical physics!

(9) $$x' = x - v\tau\gamma \ . \ y' = y \ . \ z' = z \ . \ x = x' - v'\tau\gamma$$

But is τ a time displayed on a physical clock? Take the derivative of τ, and see what we get:

$$d\tau \equiv dt - \frac{dx}{v}(1-\gamma^{-1}) \equiv dt' - \frac{dx'}{v'}(1-\gamma^{-1})$$

(10) $$d\tau \underset{dx\,=\,v\,dt}{=} dt/\gamma = \sqrt{dt^2 - dx^2} = \sqrt{dt'^2 - dx'^2} = dt'/\gamma \underset{dx'\,=\,v'dt'}{=} d\tau'$$

So the clock sought for has the same rate as the master-clocks of F & F'. Further, $x' = x$ for $\tau = 0$; this proves that the new clock will agree with the two master-clocks by coincidence. These are the criteria of *clock-congruence*, or *synchrony*. Thus τ is the common time of F & F'! That such a time exists is the great *no*!-*no*! of ***SR***. But "c'est ne pas tout", to quote Poincaré: In fact, as shown by J. Winnie [1970], a simple additive adjustment of time-zero for each single slave clock suffices to ensure that all the slave-clocks distributed over the entire co-moving frames of F & F' will agree by coincidence - and will continue to do so for ever after!

From the point of view of an observer M situated precisely midway between F & F', i.e. $MF \equiv MF'$ - and such *a midway observer* and his co-moving frame *can always be constructed* - it is evident both that the master-clocks of F & F' are in perfect synchrony, $d\tau = d\tau'$, and that their co-moving slave-clocks, after adjustment of their time-zeros, keep exactly the same rate. Hence, *to make clocks in uniform collinear motion tick in unison is not a question of changing their clock mechanisms, but only a question of adjusting their time-zeros properly*! Apparently the trick cannot be performed with more than one pair of frames at a time. But look at this:

Consider the case of three or more observers $F, F', F'', ...$ in uniform collinear motion. If they coincide at the same event, $t = t' = t''$, we can always devise an adjustment of time-zeros for their slave-clocks in order to make them all agree, without changing their mechanisms. In fact, we need no more than a single slave-clock in each fix-point of their co-moving frames if each of these clocks serves as time-keeping mechanism for an unlimited number of pointers, each of these pointers being adjusted with its own time-zero, depending on the relative velocity of that observer with respect to whose co-moving slave-clocks it is intended to agree.

Now consider instead a triply infinite (∞^3) set of observer-particles, or particle-observers. Assume that the structure of the whole set is defined by the following property: for any non-collinear triple of particle-observers belonging to the set there is a fourth one, member of the set, which is the mid-way particle of the first three, so that it remains equidistant from those three. My conjecture, then, is that such a set constitutes a substratum of fundamental observers in the sense of Milne, thus fulfilling his specific formulation of the cosmological principle ***(CP)***.

This being the case all proper distances $\mathcal{R}$ between the members of such a substratum will be subject to the same scale function $\mathcal{S}(\tau)$, with the same time τ as argument. Surprisingly, this makes the substratum an inhomogeneous *centroid* although it be everywhere *isotropic*, cf.§7.

4. FROM "BIG BANG" TO "STEADY STATE"

As we have already seen, Walker rendered the basic invariant of Milne's ***KR*** thus:

$$d\mathcal{T}^2 \;=\; dt^2 - dr^2 \;=\; d\tau^2 - \tau^2 d\sigma^2$$

This he generalized to the standard metric of modern cosmology ***(RWM)***, cf. North [1965]:

$$d\mathcal{T}^2 \;=\; d\tau^2 - \mathcal{S}^2(\tau)\, d\sigma^2 = \mathit{invar}.$$

Here τ is a universal parameter, argument in the scale factor $\mathcal{S}(\tau)$, and for $d\sigma = 0$ identical to the invariant cosmic time $\mathcal{T}$ which is the common proper time read off the master-clocks of all so-called fundamental observers, members of the substratum. For $d\sigma = 0$ we obtain:

(11) $$\underline{\sigma = \int_{\tau_1}^{\tau_2} \frac{d\tau}{\mathcal{S}(\tau)} = \int_{\tau_2}^{\tau_3} \frac{d\tau}{\mathcal{S}(\tau)} = \mathit{const.}}$$

Walker saw σ as a fixed "co-moving" coordinate characterizing a fundamental observer relative to another fundamental observer, their proper distance being defined as $\mathcal{R}(\tau) \equiv \mathcal{S}(\tau)\sigma$. The curvature of 3-space, which is latent in the element $d\sigma$, he left undetermined to start with. A time scale which eliminates the spatial expansion by the factor $\mathcal{S}(\tau)$, so that all distances between the fundamental observers remain constant, is definable by $\mathrm{T} = \int d\tau/\mathcal{S}(\tau) + \mathit{const}$. When we know the form of the scale-factor $\mathcal{S}(\tau)$, we can deduce the cosmological red-shift, $1+z(\tau) \equiv \mathcal{R}(\tau_3)/\mathcal{R}(\tau) = \mathcal{S}(\tau_3)/\mathcal{S}(\tau)$, from $\sigma = const.$ in eq.(11) for any standard ***RWM***.

The short terms, "big bang" and "steady state", were both coined by Hoyle. The first real "big bang" model was devised by a catholic priest, Lemaître, who spoke of a "primeval atom". Milne preferred to speak of *a transcendent point-event*. According to the Friedmann-Lemaître equations of General Relativity ***(GR)***, the ***BB*** model of Milne and the ***SS*** model of Bondi & Gold are both devoid of matter. A tacit presupposition, of course, is the general validity of the field equations of ***GR***. But neither Milne, nor Bondi & Gold, accepted those equations.

In fact, ***KR*** was meant by Milne to replace both ***SR*** & ***GR***. With ***KR***, Milne devised his own very ingenious theory of gravitation. As I have argued in an earlier paper, Wegener [2000], his idea can be described as "turning Mach's principle upside down": instead of trying in vain to explain inertia by reducing it to a kind of gravitation, he proposed to explain gravitation as a kind of inertia; this he did by reducing it to local deviations from global (cosmic) symmetry. From this point of view, a cosmological model is primarily described by its scale function, so there is no question of a global gravitational field acting as a "brake" on universal expansion, hence also not of taking the universe to be filled up with dark anti-gravitational energy.

The analogy between $d\tau^2 = dt^2\gamma^{-2} = dt^2 - dr^2$ and $d\mathcal{T}^2 = d\tau^2 - \mathcal{S}^2(\tau)d\sigma^2$ has been subject to conjecture by G.J. Whitrow [1961], another student of Milne's, who suggested that ***RWM*** may be derivable from the strong ***LP*** of ***SR***. However, his procedure is not convincing. Moreover, it turns out that some interesting metrics, viz. that of the "steady state" universe of Bondi & Gold, and those of two other kindred models, are in conflict with his assumption.

These models - as defined by the scale factors $\mathcal{S}_1(\tau) \equiv e^\tau$, $\mathcal{S}_2(\tau) \equiv sh\tau$, $\mathcal{S}_3(\tau) \equiv ch\tau$ - present us with the choice between preserving the standard ***RWM*** form and discarding ***LP*** as a principle of universal validity, or preserving the universal validity of ***LP*** and discarding ***RWM***. Instead of following Bondi & Gold by choosing the first option, I shall prefer the second one.

5. NEW TRANSFORMATIONS OF COORDINATES

A fundamental particle constitutes ***the natural origo*** of its own co-moving standard rest frame, the rate of its master-clock representing ***true universal time***, just as the slave-clock of an accidental particle moving in the substratum will go slow relative to true universal time.

We begin by adopting a strong differential "light principle": $d\mathcal{T}^2 = dt^2 - dr^2 = invar$. This is not meant to preclude deviations from $c_o = 1$ in peculiar experimental circumstances. A specific dependence of velocity on distance holds for all ***SS***-models. So let us assume:

(12) $$\underline{d\boldsymbol{r} = \boldsymbol{v}\,dt = th\,\boldsymbol{r}\,dt = th\,\boldsymbol{r}\,(\gamma\,d\tau) = sh\,\boldsymbol{r}\,d\tau}$$

Let $\boldsymbol{r}_{FF'} \equiv -\boldsymbol{r}_{F'F}$ be the frame distance between two fundamental observers $F\,\&\,F'$, and let $\boldsymbol{v}_{FF'} \equiv d\boldsymbol{r}_{FF'}/dt \equiv th\,\boldsymbol{r}_{FF'} \equiv -th\,\boldsymbol{r}_{F'F} \equiv -d\boldsymbol{r}_{F'F}/dt' \equiv -\boldsymbol{v}_{F'F}$ be their relative velocities. With $c \equiv$ *unity*, $\boldsymbol{v}_{FF'} \equiv th\,\boldsymbol{r}_{FF'} \simeq const.$, $\boldsymbol{LT}'$ (the differential $\boldsymbol{LT}$, cf. pp.57 & 68) become:

$$\underline{dt' = dt\,ch\,r_{FF'} - d\boldsymbol{r}\,sh\,\boldsymbol{r}_{FF'}} \ \ . \ \ \underline{d\boldsymbol{r}' = d\boldsymbol{r}\,ch\,r_{FF'} - dt\,sh\,\boldsymbol{r}_{FF'}}$$

The hyperbolic addition formulae corresponding to Galilean addition $\boldsymbol{r}+\boldsymbol{r}' \equiv \boldsymbol{r}_{FF'}$ are:

$$ch\,r' = ch\,r\,ch\,r_{FF'} - sh\,\boldsymbol{r}\,sh\,\boldsymbol{r}_{FF'} \ \ . \ \ sh\,\boldsymbol{r}' = sh\,\boldsymbol{r}\,ch\,r_{FF'} - ch\,r\,sh\,\boldsymbol{r}_{FF'}$$

It is interesting that $\boldsymbol{LT}'$ are derivable from these addition formulae if, and only if, $d\tau \equiv d\tau'$ in:

$$dt'/d\tau' \equiv ch\,r' \ \ . \ \ d\boldsymbol{r}'/d\tau' \equiv sh\,\boldsymbol{r}' \ \ . \ \ dt/d\tau \equiv ch\,r \ \ . \ \ d\boldsymbol{r}/d\tau \equiv sh\,\boldsymbol{r}$$

The $\boldsymbol{LT}'$ for three accelerated frames, $\Phi, F\,\&\,F'$, in collinear motion can be written:

$$d\boldsymbol{R} \equiv d\boldsymbol{r}\,ch\,r_{FF'} - dt\,sh\,\boldsymbol{r}_{FF'} = d\boldsymbol{r}'ch\,r_{F'F} - dt'sh\,\boldsymbol{r}_{F'F}$$
$$dT \equiv dt\,ch\,r_{FF'} - d\boldsymbol{r}\,sh\,\boldsymbol{r}_{FF'} = dt'ch\,r_{F'F} - d\boldsymbol{r}'sh\,\boldsymbol{r}_{F'F}$$

By eliminating the irrelevant frame times dt & dt' from the expressions for $d\boldsymbol{R}\,\&\,dT$ we obtain:

$$d\boldsymbol{R} = d\boldsymbol{r}/ch\,r_{\mathrm{FF'}} - dT\,th\,\boldsymbol{r}_{\mathrm{FF'}} = d\boldsymbol{r}'/ch\,r_{\mathrm{F'F}} - dT\,th\,\boldsymbol{r}_{\mathrm{F'F}}$$

With $\boldsymbol{r}_{FF'} = \boldsymbol{r}+\boldsymbol{r}' = -\boldsymbol{r}_{F'F}$, disregarding $d\boldsymbol{R}$, we obtain $\boldsymbol{LT}'$ for the common Φ-time T:

$$\underline{d\boldsymbol{r}' = \{d\boldsymbol{r}\,ch|\boldsymbol{r}_{FF'}-\boldsymbol{r}| - dT\,sh\,\boldsymbol{r}_{FF'}\}/ch\,r}$$
$$\underline{d\boldsymbol{r} = \{d\boldsymbol{r}'ch|\boldsymbol{r}_{F'F}-\boldsymbol{r}'| - dT\,sh\,\boldsymbol{r}_{F'F}\}/ch\,r'}$$

Next, introducing non-standard frame-times $d\bar{\tau}$ & $d\bar{\tau}'$ for frames $F\,\&\,F'$ defined by means of

$$\underline{d\bar{\tau} \equiv dT/ch\,r} \ \ . \ \ \underline{d\bar{\tau}' \equiv dT/ch\,r'}$$

using $\boldsymbol{v}_{FF'} \equiv th\,\boldsymbol{r}_{FF'}$, we get $\boldsymbol{TT}'$ (the differential Tangherlini trsf.s), as generalized by Selleri:

$$d\boldsymbol{r}' = d\boldsymbol{r}\,\frac{ch|\boldsymbol{r}_{FF'}-\boldsymbol{r}|}{ch\,\boldsymbol{r}} - d\bar{\tau}\,sh\,\boldsymbol{r}_{FF'} = \frac{d\boldsymbol{r}\,(1-\boldsymbol{v}_{FF'}\boldsymbol{v}) - \boldsymbol{v}_{FF'}d\bar{\tau}}{\sqrt{(1-v^2_{FF'})}}$$
$$d\boldsymbol{r} = d\boldsymbol{r}'\frac{ch|\boldsymbol{r}_{F'F}-\boldsymbol{r}'|}{ch\,r'} - d\bar{\tau}'sh\,\boldsymbol{r}_{F'F} = \frac{d\boldsymbol{r}'(1-\boldsymbol{v}_{F'F}\boldsymbol{v}') - \boldsymbol{v}_{F'F}d\bar{\tau}'}{\sqrt{(1-v^2_{F'F})}}$$

In standard ***SR***, it is always the proper time of a single moving clock which is said to be retarded relative to the slave clocks distributed as a network over the rest frame of the observer; but referring the inertial motion of particles to frame-times $d\bar{\tau}\,\&\,d\bar{\tau}'$ we must use $\boldsymbol{TT}'$ instead.

Notice that $\boldsymbol{TT}'$ reduce to $\boldsymbol{GT}'$ if measurements are referred to *the midway observer* M:

$$\boldsymbol{r} = \tfrac{1}{2}\boldsymbol{r}_{FF'} = -\boldsymbol{r}' \ \Rightarrow \ d\boldsymbol{r} - d\boldsymbol{r}' = d\bar{\tau}\,sh\,\boldsymbol{r}_{FF'} = 2\,dT\,sh\tfrac{1}{2}\boldsymbol{r}_{FF'}$$

Putting $dt \equiv d\bar{\tau} - d\boldsymbol{r}\,th\tfrac{1}{2}\boldsymbol{r}_{FF'}$, $dt' \equiv d\bar{\tau}' - d\boldsymbol{r}'\,th\tfrac{1}{2}\boldsymbol{r}_{F'F}$ directly into $\boldsymbol{LT}'$ we get the same result:

$$\underline{d\bar{\tau}' = d\bar{\tau}} \ \ . \ \ \underline{d\boldsymbol{r}' = d\boldsymbol{r} - d\bar{\tau}\,sh\boldsymbol{r}_{FF'}}$$

What has been presented above is just meant as a draft to outline some basic ideas, since a full 1x3-dim. exposition would involve the analytic hyperbolic geometry of Ungar [2008].

6. THE COSMOLOGICAL PRINCIPLE OF MILNE

Let us state a few definitions, unprimed entities referring to O and primed ones to O':

$$v^2 \equiv v_x^2+v_y^2+v_z^2 \;\; . \;\; v'^2 \equiv v_x'^2+v_y'^2+v_z'^2$$
$$v \equiv dr/dt \, , \; v_x \equiv dx/dt \;\; . \;\; v_y \equiv dz/dt \, , \; v_z \equiv dz/dt$$
$$v' \equiv dr'/dt' \, , \; v_x' \equiv dx'/dt' \;\; . \;\; v_y' \equiv dz'/dt' \, , \; v_z' \equiv dz'/dt'$$

Making use of ***LT'***, Milne calculated the velocity distribution of particles in a kinematic substratum as it is displayed to two fundamental observers, O & O', "at rest" in the substratum. Since O & O' both observe the same set of objects, viz. the substratum, they must agree about:

(13) $$f_o(v_x, v_y, v_z)\, dv_x dv_y dv_z = f_{o'}(v_x', v_y', v_z')\, dv_x' dv_y' dv_z'$$

The ***CP***, which is taken to hold for all fundamental observers, members of the substratum, but neither for arbitrary objects, nor for accidental observers not belonging to the substratum, can be viewed as a strong universal ***RP*** supporting the definability of an invariant cosmic time. Milne himself interpreted ***CP*** as a principle stating the formal identity of the functions f_o & $f_{o'}$:

(14) $$\underline{f_o \equiv f_{o'} \equiv f}$$

In order to investigate the consequences of this identification he introduced ***LT'***:

$$dx' = \frac{dx - v_{oo'}dt}{\sqrt{1-v_{oo'}^2}} \;\; . \;\; dy' = dy \;\; . \;\; dz' = dz \;\; . \;\; dt' = \frac{dt - v_{oo'}dx}{\sqrt{1-v_{oo'}^2}}$$
$$v_x' = \frac{v_x - v_{oo'}}{1-v_x v_{oo'}} \;\; . \;\; v_y' = \frac{v_y(1-v_{oo'}^2)}{1-v_y v_{oo'}} \;\; . \;\; v_z' = \frac{v_z(1-v_{oo'}^2)}{1-v_z v_{oo'}}$$

Applying partial differentiation to ***LT'*** he derived the following provisional results [1948,§52]:

$$f(v_x, v_y, v_z) = f(v_x', v_y', v_z')\frac{\partial v_x' \partial v_y' \partial v_z'}{\partial v_x \partial v_y \partial v_z}$$
$$\frac{\partial v_x' \partial v_y' \partial v_z'}{\partial v_x \partial v_y \partial v_z} = \frac{(1-v_{oo'}^2)^2}{(1-v_x v_{oo'})^4}$$
$$f(v_x, v_y, v_z) = f\left(\frac{v_x - v_{oo'}}{1-v_x v_{oo'}}, \frac{v_y(1-v_{oo'}^2)}{1-v_y v_{oo'}}, \frac{v_z(1-v_{oo'}^2)}{1-v_z v_{oo'}}\right)\frac{(1-v_{oo'}^2)^2}{(1-v_x v_{oo'})^4}$$

The most general solution of these functional equations Milne [1935] showed to be:

$$f(v_x, v_y, v_z)\, dv_x dv_y dv_z = B\,\gamma^4\, dv_x dv_y dv_z$$
$$\gamma \equiv \frac{1}{\sqrt{1-v_x^2-v_y^2-v_z^2}} \;\; . \; B = const.$$

Expressed in polar coordinates, with $d\omega$ denoting a small solid angle, his result can be written:

(15) $$\underline{f(v,\omega)\, v^2 dv\, d\omega = B\,\gamma^4\, v^2 dv\, d\omega}$$

Now, passing from this velocity distribution to the corresponding positional distribution, Milne applied the basic property of his uniformly expanding model, viz. the constancy of the relative velocities between all fundamental observers (members of the subtratum) pairwise. But at this point we must deviate from his procedure, the relative velocities between pairs of fundamental observers in our ***SS***-model being no longer constant, but increasing with distance. Hence, in order to proceed, we shall exploit the property $v \equiv dr/dt = th\, r$ implied by:

(16) $$\boxed{\underline{d\mathcal{T} = dt/ch\, r = dr/sh\, r = invar.}}$$

7. A NEW MODEL OF CONTINUED CREATION *(CC)*

The basic equations at the end of §6 are easily integrated, so we just present the result:

(17) $$\underline{\mathcal{T} = t - ln\,ch^2\tfrac{r}{2} + C = ln[2]th\tfrac{r}{2} + C}$$

Using the definitions: $\rho \equiv \mathcal{R}/e^{\mathcal{T}} \equiv [2]\,th\frac{r}{2}/e^{\mathcal{T}} \equiv$ *const.*, a natural calibration of units implies:

$$r = r_o \equiv 1 \;\Leftrightarrow\; \mathcal{R} = 2\,th\tfrac{1}{2} \;\Leftrightarrow\; t = \mathcal{T} + ln\,ch^2\tfrac{1}{2}$$

$$\mathcal{R} = \mathcal{R}_o \equiv 1 \;\Leftrightarrow\; r = 2\,arth\tfrac{1}{2} \;\Leftrightarrow\; t = \mathcal{T} - ln(1 - th^2\tfrac{r}{2}) = \mathcal{T} + ln\tfrac{4}{3}$$

If we consider a series of different fundamental particles, we get $d\rho \neq 0$, and, by differentiation:

$$e^{\mathcal{T}} d\rho = d\mathcal{R} - \mathcal{R}\,d\mathcal{T} \;\;.\;\; d\mathcal{T} = dt - th\tfrac{r}{2}\,dr$$

$$dr' \equiv\; e^t d\rho = ch\,r\,dr - sh\,r\,dt = dr - sh\,r\,d\mathcal{T}$$

(18) $$d\mathcal{T} \underset{d\rho\,=\,0}{\equiv} d\mathcal{R}/\mathcal{R} = dr/sh\,r = dt/ch\,r = dt/\gamma = \textit{invar.}$$

This is exactly what we would expect to characterize a true model of continued creation. Using $v = th\,r = \mathcal{R}/(1+\frac{\mathcal{R}^2}{4})$ to interpret Milne's velocity-distribution formula we finally get:

(19) $$\underline{B\,\gamma^4\,v^2 dv\,d\omega \;= B\,sh^2 r\,dr\,d\omega = B\,\mathcal{R}^2 d\mathcal{R}\,d\omega/(1-\tfrac{\mathcal{R}^2}{4})^3}$$

Such then is the spatial distribution of particles in a ***CC***-substratum respecting the ***CP*** of Milne. This substratum emerges as a *centroid* having *center everywhere* and *circumference nowhere*.

The *cosmic red-shift* is found to be the *standard* Doppler shift e^r, *corrected* by $e^{t-\tau}$:

(20) $$\underline{1{+}z(t) \equiv \mathcal{R}(\tau_3)/\mathcal{R}(t) = e^{\tau_3 - t} = e^{r(t)}}$$

$$1{+}z(\tau) \equiv \mathcal{R}(\tau_3)/\mathcal{R}(\tau) = e^{\tau_3-\tau} = e^{\tau_3-t}e^{t-\tau} = e^{r(t)}ch^2\tfrac{r(t)}{2} = e^{r(\tau)} = \tfrac{1+\frac{1}{2}\mathcal{R}(\tau)}{1-\frac{1}{2}\mathcal{R}(\tau)}$$

$$\underline{r = r_o \;\equiv 1 \Leftrightarrow 1{+}z(t) = e} \;\;:\;\; \underline{\mathcal{R} = \mathcal{R}_o \;\equiv 1 \;\Leftrightarrow 1{+}z(\tau) = 4}$$

Our eq.(19) is easily changed into *a number-redshift relation* by means of $\mathcal{R} = 2(1-\frac{1}{\sqrt{s}})$:

(21) $$\underline{N/N_o \propto \sqrt{1{+}z}\,(\sqrt{1{+}z}{-}1)^2\,dz/(2\sqrt{1{+}z}{-}1)^3}$$

So far our model fulfils the ***dimensional postulate*** *of Milne* [1948, §72]: *no dimensional constant of nature should be allowed to enter the definition of the kinematic substratum.*

Milne made a very illuminating distinction between the universe as ***world-map*** and the universe as ***world-view*** (literally, he spoke of 'world-picture', but my notion is slightly different). *Perceived as momentary appearance, the universe presents itself to the observer as world-view. Defined as simultaneous co-existence, the real universe should be reconstructed as world-map.*

As *world-map* our ***SS***-model has almost the same structure as the ***BB***-model of Milne. As *world-view* the two models will appear rather different to the observer. The reason is that an ***SS***-world is stationary while a ***BB***-world is evolving. For an ***SS***-world it holds that, structurally: *w.-view* $=$ *w.-map*, whereas, for a ***BB***-universe, the structures differ: *w.-view* $\neq$ *w.-map*.

The World-Map of our new CC-universe unfolds the-universe-as-it-is-in-itself as a hyperboloid in infinite 3-space - a transcendent unity of simultaneous co-existence:

(22) $$\underline{\underline{d\mathcal{T}^2 = dt^2 - ds^2}} = \textit{invariant} \;\;.\;\; ds^2 = dr^2 + r^2\,(d\theta^2 + sin^2\theta d\phi^2)$$ **- 2015**

The World-View of our new CC-universe depicts the-universe-as-it-appears-to-us as a pseudo-sphere in finite 3-space - an observable reality of shells of increasing age:

(23) $$\underline{\underline{dt^2 = d\mathcal{T}^2 + ds^2}} \;\;.\;\; ds^2 = \frac{d\mathcal{R}^2}{(1-\frac{\mathcal{R}^2}{4})^2} + arsh\frac{\mathcal{R}^2}{(1-\frac{\mathcal{R}^2}{4})^2}(d\theta^2 + sin^2\theta\,d\phi^2)$$ **- 2015**

Compare the interesting analysis of the "cone-model" in Osborne & Pope [2007].

8. TWO ASYMPTOTIC APPROXIMATIONS

The Model M_2 *approximates our **CC**-model of continuous creation from a "**Fiery Blow**".* Model M_2 is constructed in the following way: Adopting LT' - the differential Lorentz Group as entailed by the strong ***LP*** - we assume these time dependent velocity-distance relations:

(24)
$$\underline{\underline{\rho \equiv sh\,r/sh\,t \equiv 2\,th\tfrac{r}{2}/sh\,\tau}}$$
$$sh\,t\,d\rho = ch\,r\,dr - sh\,r\,dt/th\,t = dr - sh\,r\,d\tau/th\,\tau$$
$$v \equiv dr/dt \underset{d\rho\,=\,0}{=} thr/th\,t = th\,r\,\frac{\sqrt{1+sh^2t}}{sh\,t} = \frac{\sqrt{sh^2r+\rho^2}}{ch\,r}$$
$$\gamma_v \equiv \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-(sh^2r+\rho^2)/ch^2r}} = \frac{ch\,r}{\sqrt{1-\rho^2}}$$

We are now able to express our invariant cosmic time $\mathcal{T}$ as a function of r & ρ:
$$\mathcal{T} \equiv \int \gamma^{-1}dt - C = \int \frac{\sqrt{1-\rho^2}}{\sqrt{sh^2r+\rho^2}}dr - C \underset{C\,\equiv\,ln\rho}{\rightarrow} ln(2th\tfrac{r}{2}) - ln\rho \equiv ln(\mathcal{R}/\rho)$$

The cosmic red-shift is finally to be the standard Doppler shift corrected by $\frac{sh\,t}{sh\,\tau}$:
$$\rho = \frac{\mathcal{R}(t)}{sh\,t} = \frac{\mathcal{R}(\tau)}{sh\,\tau} = \frac{\mathcal{R}(\tau_3)}{sh\,\tau_3} = const.$$

(25a) $$1+z(t) = \frac{\mathcal{R}(\tau_3)}{\mathcal{R}(t)} = \frac{sh\,\tau_3}{sh\,t} = e^{\tau_3 - t}\left(\frac{1-exp\text{-}2\tau_3}{1-exp\text{-}2t}\right) \rightarrow e^{r(t)} = \frac{1+\frac{1}{2}\mathcal{R}(t)}{1-\frac{1}{2}\mathcal{R}(t)}$$

(25b) $$1+z(\tau) = \frac{\mathcal{R}(\tau_3)}{\mathcal{R}(\tau)} = \frac{sh\,\tau_3}{sh\,t}\frac{sh\,t}{sh\,\tau} \rightarrow e^{r(t)}ch\frac{r(t)}{2} = e^{r(\tau)} = \frac{1+\frac{1}{2}\mathcal{R}(\tau)}{1-\frac{1}{2}\mathcal{R}(\tau)}$$

$$\mathcal{H}_2(\mathcal{T}) \equiv \dot{\mathcal{R}}_2(\mathcal{T})/\mathcal{R}_2(\mathcal{T}) \propto cth\,\mathcal{T}.\ \Rightarrow .\ \mathcal{H}_2 \underset{\mathcal{T}\,\simeq\,0}{\simeq} \infty$$

We see that, near time zero, the velocities of dispersion far transcend that of light thereby obviating the supposed need for inflation in a very natural way.

The Model M_3 *approximates our **CC**-model of continuous creation by a "**Gentle Flow**".* Model M_3 is constructed in a similar manner: Adopting LT' - the differential Lorentz Group as entailed by the strong ***LP*** - we assume these time dependent velocity-distance relations:

(26)
$$\underline{\underline{\rho \equiv sh\,r/ch\,t \equiv 2\,th\tfrac{r}{2}/ch\tau}}$$
$$ch\,t\,d\rho = ch\,r\,dr - sh\,r\,dt\,th\,t = dr - sh\,r\,th\,\tau\,d\tau$$
$$v \equiv dr/dt \underset{d\rho\,=\,0}{=} th\,r\,th\,t = th\,r\,\frac{\sqrt{ch^2t-1}}{ch\,t} = \frac{\sqrt{sh^2r-\rho^2}}{ch\,r}$$
$$\gamma_v \equiv \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-(sh^2r-\rho^2)/ch^2r}} = \frac{ch\,r}{\sqrt{1+\rho^2}}$$

We are now able to express our invariant cosmic time $\mathcal{T}$ as a function of r & ρ:
$$\mathcal{T} \equiv \int \gamma^{-1}dt - C = \int \frac{\sqrt{1+\rho^2}}{\sqrt{sh^2r-\rho^2}}dr - C \underset{C\,\equiv\,ln\rho}{\rightarrow} ln(2th\tfrac{r}{2}) - ln\rho \equiv ln(\mathcal{R}/\rho)$$

The cosmic red-shift is finally to be the standard Doppler shift corrected by $\frac{ch\,t}{ch\,\tau}$:
$$\rho = \frac{\mathcal{R}(t)}{ch\,t} = \frac{\mathcal{R}(\tau)}{ch\,\tau} = \frac{\mathcal{R}(\tau_3)}{ch\,\tau_3} = const.$$

(27a) $$1+z(t) = \frac{\mathcal{R}(\tau_3)}{\mathcal{R}(t)} = \frac{ch\,\tau_3}{ch\,t} = e^{\tau_3 - t}\left(\frac{1+exp\text{-}2\tau_3}{1+exp\text{-}2t}\right) \rightarrow e^{r(t)} = \frac{1+\frac{1}{2}\mathcal{R}(t)}{1-\frac{1}{2}\mathcal{R}(t)}$$

(27b) $$1+z(\tau) = \frac{\mathcal{R}(\tau_3)}{\mathcal{R}(\tau)} = \frac{ch\,\tau_3}{ch\,t}\frac{ch\,t}{ch\,\tau} \rightarrow e^{r(t)}ch\frac{r(t)}{2} = e^{r(\tau)} = \frac{1+\frac{1}{2}\mathcal{R}(\tau)}{1-\frac{1}{2}\mathcal{R}(\tau)}$$

$$\mathcal{H}_3(\mathcal{T}) \equiv \dot{\mathcal{R}}_3(\mathcal{T})/\mathcal{R}_3(\mathcal{T}) \propto th\,\mathcal{T}.\ \Rightarrow .\ \mathcal{H}_3 \underset{\mathcal{T}\,=\,0}{=} 0$$

We see that, at time zero, the entire universe is completely stationary.

9. CONSIDERATIONS OF ENERGY

The kinematic substratum functions as a *compass of inertia* (Weyl, Gödel) by defining the states of rest and motion in the universe. While fundamental observers may be considered to be locally at rest, all other particles not members of the substratum - let us call them accidental - are distinguished by their motion. Now the substratum is dense, cf. the midway property of §3. Given that an accidental particle A is passing a fundamental observer O with velocity $\vec{v}_{oa} \equiv \vec{v}_{oo'}$ at instant t_o, we have by the same token also found that other fundamental observer O' relative to which it is instantaneously at rest, their distance being $\vec{r}_{ao'} \equiv \vec{r}_{oo'}$ at the same instant t_o.

Hence the instantaneous state of motion of an accidental particle is fully specified by two fundamental observers: that with which it coincides, and that with respect to which it is at rest. Now, according to ***CP****, all fundamental particles are equivalent. Granted that the energy in any volume of fixed size, is constant, cf. the energy principle (****PCE****), and assuming the classical equivalence of velocity of escape to gravitational potential,* $v_\infty^2 = -2\varphi = Gm_o/r$*, what appears as the kinetic energy of* A *relative to* O*,* $\boldsymbol{T}_{oa} = m_a\gamma_{\vec{v}} = m_a/\sqrt{1-\vec{v}^2}$*, must likewise appear as the dynamic, or potential, energy of* A *relative to* O'*,* $-\boldsymbol{V}_{o'a} = m_a\gamma_{\vec{\varphi}} = m_a/\sqrt{1+2\vec{\varphi}}$*. So we shall take* $\gamma_{\boldsymbol{v}} - \gamma_{\boldsymbol{\varphi}}$ *to be a basic constant for fundamental observers in accordance with:*

$$\boldsymbol{T} = m_o[c^2](\gamma_{\vec{v}}-1) = m_{\vec{v}} - m_o \tag{28}$$

$$-\boldsymbol{V} = m_o[c^2](\gamma_{\vec{\varphi}}-1) = m_{\vec{\varphi}} - m_o \tag{29}$$

$$\begin{aligned} d\boldsymbol{T} &= \boldsymbol{F}\,dr = (\tfrac{dp}{dt})dr = \tfrac{d}{dt}(m_o\gamma_v v)dr \\ &= m_o\{v\tfrac{d\gamma_v}{dt} + \gamma_v\tfrac{dv}{dt}\}dr = m_o\{v^2 d\gamma_v + \gamma_v v dv\} \\ &= m_o\{v^2 d\gamma_v + \gamma_v^{-2} d\gamma_v\} = m_o[c^2]d\gamma_v = dm_{\vec{v}} \\ -d\boldsymbol{V} &= \boldsymbol{F}\,dr = \{\tfrac{d}{dr}(m_o\gamma_\varphi)\}dr = m_o[c^2]d\gamma_\varphi = dm_{\vec{\varphi}} \end{aligned}$$

$$\boldsymbol{H} \equiv \boldsymbol{T} + \boldsymbol{V} = m_{\vec{v}} - m_{\vec{\varphi}} = m_o(\gamma_{\vec{v}} - \gamma_{\vec{\varphi}}) = const. \tag{30}$$

$$\boldsymbol{L} \equiv \boldsymbol{T} - \boldsymbol{V} = m_{\vec{v}} + m_{\vec{\varphi}} - 2m_o = m_o(\gamma_{\vec{v}} + \gamma_{\vec{\varphi}} - 2) \tag{31}$$

Further, assuming the principle of least action (***PLA***), and using the above Lagrangian, we get *a variational principle describing the observed perihelion displacement of Mercury*:

$$\delta\int_{t_1}^{t_2}\boldsymbol{L}\,dt = \delta\int_{t_1}^{t_2}(m_{\vec{v}} + m_{\vec{\varphi}})dt = 0 \quad\Rightarrow\quad \frac{d}{dt}\left(\frac{\partial m_{\vec{v}}}{\partial \dot{q}_i}\right) - \frac{\partial m_{\vec{\varphi}}}{\partial q_i} = 0 \tag{32}$$

Following Prokhovnik [1988], a unit-rod moving in a substratum ("ether") is reduced by:

$$c_{v,\theta}^{-1} = \tfrac{1}{2}(c_{\rightarrow}^{-1} + c_{\leftarrow}^{-1}) = \frac{\sqrt{1-v^2 sin^2\theta}}{1-v^2} \tag{33}$$

Hence, due to the local asymmetry introduced by the motion, the longitudinal speed of a photon will be $c_{v,0} = 1-v^2$, its transversal speed beng $c_{v,\frac{\pi}{2}} = \sqrt{1-v^2}$. By analogy we then assume:

$$c_{v,\theta}^{-1} = \frac{\sqrt{1-v^2 sin^2\theta}}{1-v^2} \;\Rightarrow\; c_{\varphi,\theta}^{-1} = \frac{\sqrt{1+2\varphi sin^2\theta}}{1+2\varphi} \tag{34}$$

Now $\delta\int_{t_1}^{t_2}\frac{2r}{c_\varphi}dt$ for $\theta \simeq 0$ yields *the observed delay of light-signals* reflected from a planet while *the observed deflection of light rays* near a massive body is found by a Fermat principle:

$$\delta\int_{t_1}^{t_2}\frac{dr}{c_\varphi} = \delta\int_{r_1}^{r_2}\frac{\sqrt{1+2\varphi sin^2\theta}}{1+2\varphi}dr = 0 \tag{35}$$

O. CONCLUSION

For astronomical purposes, *the concept of light-time distance* r is particularly convenient. In this measure, which relates to frame-time t, the Hubble function takes on the non-linear form:

$$\boxed{v/r \underset{t\to\infty}{\longrightarrow} th\, r/r \underset{r\equiv 1}{\equiv} th\, 1}$$

This is a very definite prediction. Expressed in proper distance $\mathcal{R}$ and proper time $\mathcal{T}$ it becomes:

$$\boxed{d\mathcal{R}/\mathcal{R}\, d\mathcal{T} \underset{\mathcal{T}\to\infty}{\longrightarrow} \textit{unity}}$$

Another prediction is that the velocity-distribution in a substratum tends towards a steady state:

$$\boxed{B\, \gamma^4\, v^2 dv\, d\omega \underset{t\to\infty}{\longrightarrow} B\, sh^2 r\, dr\, d\omega}$$

Our ***MM***$_{1\text{-}3}$ notion of proper distance, $\mathcal{R} \equiv 2\, th\frac{r}{2} \rightarrow e^{\mathcal{T}}\rho$, looks strange to most people. But it has a simple explanation, being the distance between two fundamental observers, O & O', as defined in the frame of a collinear midway-particle, M. Imagine $v_{om} = th\, r_{om} = th\frac{1}{2}r_{oo'}$ and $v_{o'm} = th\, r_{o'm} = th\frac{1}{2}r_{o'o}$ both approaching 1 as $r \rightarrow \infty$; then, just as $v_{om} - v_{o'm} \underset{r\to\infty}{\longrightarrow} 2$, we have:

$$\boxed{\mathcal{R}_{oo'} = 2\, th\tfrac{1}{2}r_{oo'} = thr_{om} + thr_{mo'} \rightarrow 2}$$

Thus, while light-time distance r is simply additive, proper distance $\mathcal{R}$ adds up like $2\, th\frac{r}{2}$. This means that ***MM***$_{1\text{-}3}$ display a density increasing with distance both according to world-view and according to world-map, i.e., ***MM***$_{1\text{-}3}$ can only be perceived and described in a perspective. In this way our models ***MM***$_{1\text{-}3}$ conform to the description by Nicholas of Cusa [ca.1450]:

The world machinery is as if it had its center everywhere and circumference nowhere, its circumference and center being no other than God who is everywhere and nowhere.

From every possible point of view, defined by reference to a single fundamental observer, the universe is interpretable as a steady stream - confined within the limits of a sphere - which may be stationary (our "*Steady State*" model ***M***$_1$), exploding from a transcendent singularity (our "*Fiery Blow*" model ***M***$_2$) or emanating from a static state (our "*Gentle Flow*" model ***M***$_3$).

All models ***MM***$_{1\text{-}3}$ are non-Friedmannian and conform to Milne's no-horizon principle. So they avoid the usual problems associated with the standard ***RW*** models (flatness, horizons). There is no reason to invoke the idea of inflation. Further, being comprised within the confines of a pseudo-sphere with radius $\mathcal{R}_o = 2$, a constant universal mass and energy is definable.

It is surprising that a receding particle will reach the universal border at $\mathcal{R}_o = 2$ within the finite time $\Delta\mathcal{T} \simeq ln(2/\rho)$. We shall say that, at this instant, the particle leaves the universe, it no longer exists; thus everything which can properly be said to exist does so within the world, understood as the *world-view*. The energy of this pseudo-sphere must be constant; zero, say.

With our world ***M***$_1$, we are confronted with the picture of an eternal universe where the energy lost at the periphery is compensated by a steady gain in energy at the center, from which a steady aethereal stream of matter and light is pouring out towards all possible directions.

MM$_{2-3}$ are just different approximations to the same overall view of continuous creation. For this reason our new "Milne" models ***MM***$_{1\text{-}3}$ can be claimed to represent the true synthesis of the opposite cosmological views of the ancient philosophers *Parmenides* and *Herakleitos*.

=//=

Mogens True Wegener

Appendix: $WW_{0\text{-}3}$ as based on RWM

Milne's BB-model $M_0 \equiv$ Walker's BB-model W_0

$$\mathcal{S}_0(\tau) \equiv \tau \ . \ \int \tfrac{d\tau}{\tau} + C$$
$$\tau \equiv \tfrac{d\tau}{d\mathrm{T}} = e^{\mathrm{T}-1} \Rightarrow . \ \tau = \left\{ {}^{\infty}_{\substack{1\\0}} \Leftrightarrow \mathrm{T} = \left\{ {}^{\infty}_{\substack{1\\-\infty}} \right.\right.$$
$$d\rho \propto \tau^{-1} d\tau \ \Rightarrow \ \rho = [ln\tau]^{\tau_2}_{\tau_1} = [ln\tau]^{\tau_3}_{\tau_2} = const.$$
$$t = \tfrac{1}{2}(\tau_3 + \tau_1) = \tau\, ch\rho \ \Rightarrow \ dt = d\tau\, ch\rho + \tau\, d\rho\, sh\rho$$
$$r = \tfrac{1}{2}(\tau_3 - \tau_1) = \tau\, sh\rho \ \Rightarrow \ dr = d\tau\, sh\rho + \tau d\rho\, ch\rho$$
$$\tfrac{dr}{dt} \underset{d\rho\,=\,0}{=} \tfrac{r}{t} = th\rho \ \Rightarrow \ \gamma \equiv \tfrac{1}{\sqrt{1 - dr^2/dt^2}} \underset{d\rho\,=\,0}{=} \tfrac{1}{\sqrt{1 - th^2\rho}} = ch\rho$$
$$d\mathcal{T}^{\,2} \equiv dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2) =$$
$$= d\tau^2 - \tau^2\{d\rho^2 + sh^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\} =$$
$$= e^{2(\mathrm{T}-1)}\{d\mathrm{T}^2 - d\rho^2 - sh^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\}$$

The Walker worlds $WW_{1\text{-}3}$ are presented in the usual way:
*as derived from their scale functions by means of the **RWM***
*in a form incompatible with **LT′** and the strong version of **LP**.*

The original SS ("Steady State") model W_1 of Bondi & Gold

$$\mathcal{S}_1(\tau) \equiv e^{\tau} \ . \ \int \tfrac{d\tau}{exp\,\tau} + C = -e^{-\tau} + C$$
$$e^{\tau} \equiv \tfrac{d\tau}{d\mathrm{T}} = \tfrac{1}{1-\mathrm{T}} \Rightarrow . \ \tau = \left\{ {}^{\infty}_{\substack{0\\-\infty}} \Leftrightarrow \mathrm{T} = \left\{ {}^{1}_{\substack{0\\-\infty}} \right.\right.$$
$$d\rho \propto e^{-\tau} d\tau \ \Rightarrow \ \rho = [-e^{-\tau}]^{\tau_2}_{\tau_1} = [-e^{-\mathcal{T}}]^{\tau_3}_{\tau_2} = const.$$
$$\rho = e^{-\tau_1} - e^{-\tau} = e^{-\tau} - e^{-\tau_3} \ \Rightarrow \ e^t \rho = e^r - e^{t-\tau} = e^{t-\tau} - e^{-r}$$
$$e^{t-\tau} = ch\, r \ \Rightarrow \ d\tau = dt - dr\, th\, r \ \ . \ e^t\rho = sh\, r \ \Rightarrow \ e^{\tau} d\rho = dr - dt\, th\, r$$
$$d\tau^2 - e^{2\tau} d\rho^2 = (dt^2 - dr^2)(1 - th^2 r) = (dt^2 - dr^2) ch^{-2} r$$
$$d\mathcal{T}^2 = d\tau^2 - e^{2\tau}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\} =$$
$$= \{dt^2 - dr^2 - sh\, r^2(d\theta^2 + sin^2\theta\, d\phi^2)\} ch^{-2} r$$
$$\tfrac{dr}{dt} \underset{d\rho\,=\,0}{=} th\, r \ \Rightarrow \ \gamma \equiv \tfrac{1}{\sqrt{1 - dr^2/dt^2}} \underset{d\rho\,=\,0}{=} \tfrac{1}{\sqrt{1 - th^2 r}} = ch\, r$$

The FB ("Fiery Blow") model W_2 (not the FB-model M_2):

$$\mathcal{S}_2(\tau) \equiv sh\tau \ . \ \int \tfrac{d\tau}{sh\tau} + C = ln\, th\tfrac{1}{2}\tau + C$$
$$sh\tau \equiv \tfrac{d\tau}{d\mathrm{T}} = \tfrac{1}{sh(1 - \mathrm{T} + arsh\frac{1}{sh1})} \Rightarrow . \ \tau = \left\{ {}^{\infty}_{\substack{1\\0}} \Leftrightarrow \mathrm{T} = \left\{ {}^{1 - ln th\frac{1}{2}}_{\substack{1\\-\infty}} \right.\right.$$
$$d\rho \propto sh^{-1}\tau\, d\tau \ \Rightarrow \ \rho = [ln\, th\tfrac{1}{2}\tau]^{\tau_2}_{\tau_1} = [ln\, th\tfrac{1}{2}\tau]^{\tau_3}_{\tau_2} = const.$$
$$e^{\rho} = th\tfrac{1}{2}\tau / th\tfrac{1}{2}\tau_1 = th\tfrac{1}{2}\tau_3 / th\tfrac{1}{2}\tau_2$$
$$t = \tfrac{1}{2}(\tau_3 + \tau_1) = arth(th\tfrac{1}{2}\tau\, e^{\rho}) + arth(th\tfrac{1}{2}\tau\, e^{-\rho}) = arth(th\tau\, ch\rho)$$
$$r = \tfrac{1}{2}(\tau_3 - \tau_1) = arth(th\tfrac{1}{2}\tau\, e^{\rho}) - arth(th\tfrac{1}{2}\tau\, e^{-\rho}) = arth(sh\tau\, sh\rho)$$

$$r = arth(sh\tau\, sh\rho) \;\Rightarrow\; dr = \frac{ch\tau\, d\tau\, sh\rho + sh\tau\, ch\rho\, d\rho}{1 - sh^2\tau\, sh^2\rho}$$

$$t = arth(th\tau\, ch\rho) \;\Rightarrow\; dt = \frac{ch^{-2}\tau\, d\tau ch\rho + th\tau\, sh\rho\, d\rho}{1 - th^2\tau\, ch^2\rho} = \frac{d\tau\, ch\rho + sh\tau\, ch\tau\, sh\rho\, d\rho}{1 - sh^2\tau\, sh^2\rho}$$

$$\underline{d\mathcal{T}^2 = d\tau^2 - sh^2\tau\{d\rho^2 + sh^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

$$\underline{\frac{dr}{dt}\underset{d\rho\,=\,0}{\equiv}\frac{th\, r}{th\, t}} \;\Rightarrow\; \gamma \equiv \frac{1}{\sqrt{1 - dr^2/dt^2}} \underset{d\rho\,=\,0}{\equiv} \frac{1}{\sqrt{1 - th^2 r/th^2 t}}$$

The GF ("Gentle Flow") model W_3 (not the GF-model M_3):

$$\underline{\mathcal{S}_3(\tau) \equiv ch\tau \;\;.\;\; \int \frac{d\tau}{ch\tau} + C = arsin\, th\tau + C}$$

$$ch\tau \equiv \frac{d\tau}{d\mathrm{T}} = tan\mathrm{T} \;\Rightarrow\; \tau = \left\{ \begin{matrix} \infty \\ 0 \\ -\infty \end{matrix} \right. \Leftrightarrow \mathrm{T} = \left\{ \begin{matrix} \pi/2 \\ \pi/4 \\ \pi/2 \end{matrix} \right.$$

$$\underline{d\rho \propto ch^{-1}\tau\, d\tau \;\Rightarrow\; \rho = [arsin\, th\tau]_{\tau_1}^{\tau_2} = [arsin\, th\tau]_{\tau_2}^{\tau_3} = \textit{const.}}$$

$$th\tau_3 = sin(arsin\, th\tau + \rho) \;\;.\;\; th\tau_1 = sin(arsin\, th\tau - \rho)$$

$$th2t = th(\tau_3 + \tau_1) = \frac{2\, th\tau\, cos\rho}{cos^2\rho + th^2\tau} \;\Rightarrow\; :th\, t = th\tau/cos\rho \;\Rightarrow\; dt = \frac{d\tau\, cos\rho + sh\tau\, ch\tau\, sin\rho d\rho}{1 - ch^2\tau\, sin^2\rho}$$

$$th2r = th(\tau_3 - \tau_1) = \frac{2\sqrt{1 - th^2\tau}\, sin\rho}{1 - th^2\tau + sin^2\rho} \;\Rightarrow\; :th\, r = ch\tau\, sin\rho \;\Rightarrow\; dr = \frac{sh\tau\, d\tau\, sin\rho + ch\tau\, cos\rho\, d\rho}{1 - ch^2\tau\, sin^2\rho}$$

$$\underline{d\mathcal{T}^2 = d\tau^2 - ch^2\tau\{d\rho^2 + sin^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

$$\underline{\frac{dr}{dt}\underset{d\rho\,=\,0}{\equiv}\frac{th\, r\, th\, t}{1}} \;\Rightarrow\; \gamma \equiv \frac{1}{\sqrt{1 - dr^2/dt^2}} \underset{d\rho\,=\,0}{\equiv} \frac{1}{\sqrt{1 - th^2 r\, th^2 t}}$$

All three "Walker"-models $WW_{1\text{-}3}$

$$\mathcal{R}_{AB}(\mathcal{T}) \equiv \mathcal{S}_i(\mathcal{T})\rho_{AB} \;\;.\;\; \mathrm{T} \equiv \int d\tau/\mathcal{S}_i(\tau) + C$$

$$\rho = \int_{\tau_1}^{\tau_2} \frac{d\tau}{\mathcal{S}_i(\tau)} = \int_{\tau_2}^{\tau_3} \frac{d\tau}{\mathcal{S}_i(\tau)} = \textit{const.} \Rightarrow .\; \frac{d\tau_3}{\mathcal{S}_i(\tau_3)} = \frac{d\tau_2}{\mathcal{S}_i(\tau_2)} \Rightarrow \frac{\mathcal{S}_i(\tau_3)}{\mathcal{S}_i(\tau_2)} = \frac{d\tau_3}{d\tau_2} = s$$

$$\underline{d\mathcal{T}^{\,2} = d\tau^2 - \mathcal{S}_i(\tau)\{d\rho^2 + f^2(\rho)(d\theta^2 + sin^2\theta\, d\phi^2)\} =}$$

$$d\mathcal{T} \underset{d\theta\,=\,d\phi\,=\,0}{\equiv} \sqrt{dt^2 - dr^2}/ch\, r = dt/\gamma\, ch\, r \underset{r\,\simeq\,0}{\simeq} dt/\gamma$$

$$\mathcal{T} = \tau \;\Leftrightarrow\; d\rho = d\theta = d\phi = 0 \;\;.\;\; \mathcal{T} = t \;\Leftrightarrow\; r = dr = 0$$

An Important Proviso

The difference between our "Milne" models ***MM***$_{1\text{-}3}$ and our "Walker" models ***WW***$_{1\text{-}3}$ is: For ***M***$_1$, *world-view* is identical to *world-map* in the sense that we see the visible objects as they were when they emitted their light, and their general structure then is similar to their general structure now. The world observed is potentially infinite, actual infinity reigns at the periphery; nothing is "outside", hidden by a horizon. For ***W***$_1$, this is not the case: the observable universe, being finite, is an infinitesimal drop in an infinite ocean of "reality", so an impenetrable horizon separates what we can see from what there was or is. Philosophically, this is not satisfying as it makes it impossible to state a sensible definition of the total amount of energy contained in ***W***$_1$. Finally, what applies to ***M***$_1$, resp.***W***$_1$, by approximation also applies to ***MM***$_{2\text{-}3}$, resp.***WW***$_{2\text{-}3}$.

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   "*Unde erit machina mundi quasi habens undique centrum et nullibi circumferentia, quoniam eius circumferentia et centrum est Deus qui est undique et nullibi.*"
   cf.II,xii; see also the introduction to Wegener [1999].
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