CHAPTER 7

CONSTRUCTIVISM IN SCIENCE FROM POINCARÉ TO EDDINGTON AND MILNE

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SUMMARY

The merits of Poincaré as one of the greatest mathematicians of all times are globally acknowledged, but the value of his conventionalist theory of science is still greatly underestimated, and his contributions to physics and its philosophy have unjustly fallen into oblivion. The aim of the present paper is to stress the importance of Poincaré to physical theory and the theory of physics by hailing him as the principal figure in the interplay between classical philosophy and modern cosmology.

Minor sections on Eddington and Milne are added to the main article in order to show the influence of Poincaré, albeit mostly indirect, on two major figures of British Cosmology.

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A FEW QUOTATIONS

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"Nothing of all that which has been set forth about the universe could ever have been said if we had never seen the sun or the starry heavens; but observation of day and night, of months and seasons of the year, of equinox and solstice, has produced our knowledge of numbers, which has conferred on us the notion of time and inspired us to investigate the universe; whence we have got philosophy which is the greatest boon ever bestowed on mortal man by the heavens.

The cause and purpose of vision is this: God invented it and entrusted it to us in order that we should observe the orbits of reason in the heavens and use them to correct the circuits of our thought which are akin to them, though ours be troubled and they unperturbed, so that - when we learned to know them and to compute them rightly according to nature - we could bring order to our own errant circles by imitating those of God which are perfectly regular."

Plato [22]

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"Reason does not extract its laws from nature, it prescribes them to nature ... In this way, by subsuming all phenomena under its own laws, reason is the source and origin of the general order of nature ... Simple, as is the origin of this law (of reciprocal attraction), which relies only on the relationships between spherical surfaces of different radii, nevertheless its consequences are so rich, as regards the variety of their mutual consistencies and uniformities, that not only does it describe all possible trajectories of heavenly bodies by conical sections, but it does also imply relations of such a kind to obtain between these sections, that no other law of gravitation than that depending on the inverse square of the distance can be suitable to a world system."

Kant [10]

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"Does the harmony the human intelligence thinks it discovers in nature exist outside of this intelligence? No, beyond doubt, a reality completely independent of the mind which conceives it, sees or feels it, is impossible. A world as exterior as that - even if it existed - would for us be forever inaccessible. But what we call objective reality is, in the last analysis, what is common to many thinking beings, and could be common to all. That common part can only be a harmony expressed by mathematical laws. It is this harmony, then, which is the sôle objective reality, the only truth we can attain. When I add that the universal harmony of the world is the source of all beauty, it will be understood what prize we should attach to the slow and difficult progress which little by little enables us to know it better."

Poincaré [24]

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1. FROM KANT TO PLATO

The merits of Jules Henri Poincaré as one of the greatest mathematicians of all times are globally acknowledged. But the value of his conventionalist theory of science is still seriously underestimated, partly due to misrepresentation by historians, partly due to unfair criticism propagated by philosophers; and his great contributions to physics and its philosophy have unjustly fallen into oblivion as compared to the overwhelming fame of Albert Einstein.

In the present paper it is *my aim to stress the importance of Poincaré to physical theory and the theory of physics by hailing him as* ***the*** *principal figure in the traditional interplay between classical philosophy and modern cosmology*. As an example, I want to highlight him as the central link in a line of development connecting the main stream of European thought, as represented by Kant's *Critique of Pure Reason*, to two seemingly incompatible non-standard cosmologies: viz., that of Arthur Stanley Eddington, and that of Edward Arthur Milne.

Within the restricted frame of time and space allowed to me at this very special occasion it is of course not advisable for me to dwell at length on historical detail, neither do I feel able to do so without further study. What I want to do is to draw, with coloured brush and sweeping gesture, some very broad lines in the history of scientific ideas. These, as I see it, opens some exciting philosophical perspectives that might in the end help us to throw light on the present impasse of cosmology. But in order not to pretend too much, I shall close these introductory remarks by reminding you of the obvious fact, that science is always in need of bold new ideas. This is one of the reasons why we should not forget about its history.

The story of how Immanuel Kant was disturbed in his dogmatic slumber by the doubts of an eloquent Scotchman is well-known: finally he was forced to confront the scandal of the contemporary philosophy. A century earlier, Isaac Newton had obtained to physics its most brilliant triumph ever, yet philosophy had been unable to account for this unique achievement, let alone to disclose its legitimacy. In spite of Descartes, the ghost of Aristotelianism was still haunting philosophy. But to Kant, at least, it had become clear that the way of abstraction is blocked: true knowledge can never be obtained by the process of successive approximation. Inspired by the feat of Nicholas Koppernigk, the great innovator of medieval astronomy, Kant now set himself the task to effect a Copernican revolution in philosophy.

In order to further "the safe progress of science", Kant proposed a distinction between plain phenomena and true noumena or, as he said, between *reality-for-us* and *reality-in-itself*. Of *phenomena*, belonging to the realm of experience and stemming from the joint venture of observation and speculation or from the teamwork of sensation and reflection, we can know everything. Of *noumena*, permanently hidden, we can know nothing; the inner nature of reality transcends our inborn intellectual capabilities forever. But apparent nature, the plain surface of reality, remains transparent to our intelligence. What we must do, in order to obtain absolute and indubitable knowledge, is only to apply those conceptions which distinguish our inborn nature as thinking beings from the manifold of those sensations that are continuously caught by the network defining the structure of our natural intuition: the framework of time and space. True knowledge of apparent nature, reality-for-us, is then bound to emerge.

Knowledge of this kind, albeit occasioned by experience, gets its validity and legitimacy from another source, viz., the collaboration of reason and intuition. According to Kant, pure reason can collaborate with pure intuition ahead of any mediation of experience, and the result of this activity is pure knowledge *á priori*. Knowledge is *á priori* if it can be constructed by strictly transcendental arguments, i.e., formal arguments which are valid independently of any concrete experience. As regards the *á priori* argument given by Kant to prove the inverse square law of gravitation, it is clear that his claim - astounding as it appears - can be sustained on the assumption that gravitational forces can be described in flat 3-space: his argument is then on a par with that leading to the so-called Olbers' paradox. Though we always have to wait for such arguments to be invented, it is nevertheless interesting to speculate how Kantianism might have been received if - *per impossibile* - the Euclidean parallel axiom had eventually been proven.

To Poincaré, the failure of all the proofs given hitherto was decisive evidence against the claim of Kant that the structure of space can be demonstrated *a priori*. Although accepting the possibility of a pure intuition of space, he claimed that such space is devoid of any formal structure, thus only definable in negative terms. This brought him close to the position of Plato who frankly stated that space, "the uterus of becoming", is nothing but an imaginary container whose dreamlike existence it "is hard to believe in"; formless, and causally ineffective, it is "next to nothing". Following Descartes, extension is a substantial property and space is material; but Poincaré rejected the Cartesian aether-hypothesis, just as it had been rejected by Leibniz, and that for the very same reason: abstract space is relational, not substantial.

It remains for us to point out that the gist of Kantian *á-priorism* is not bound up to the problem of geometry and, *á fortiori*, not to the idea of an aether. Its real issue is *the active rôle of the intellect* in the reconstruction of the universe. This activity of reconstructing the universe is performed by *the transcendental subject* which may be interpreted as a kind of Platonic demiurge stripped of mythology: the *demiurge* created *cósmos* from *cháos* by applying ideas of geometry and categories of logic to a pre-existing ocean of sensible qualities; in the same way the transcendental subject produces its world by applying reason to intuition.

But the most notable difference between Kant and Plato is that Plato considered physics to be "the science of the probable", not *epistéme*, but *dóxa,* whereas Kant insisted on *epistéme,* i.e., a final and absolute knowledge of nature. At this point modern science is clearly much more close to Plato than it is to Kant. Plato dreamt of a kinship between mind and nature, between concept and reality. He assumed that harmony is inherent in the world of nature, believing that it can be discovered by human reason because reason itself is part of that harmony.

I will elaborate a little more on this idea, as expressed by Poincaré, Eddington and Milne. It turns out that the modern equivalent to the ancient idea of harmony is mathematical: group-theoretical isomorphism. This is the clue to the art of world-building in modern cosmology.

2. JULES HENRI POINCARÉ (1854-1912)

In contrast to the theme of this conference, *Science et Hypothése*, which refers to the first of the four books on the philosophy of science written by Poincaré [1902], my own reflections will primarily put focus on the second of these, viz. *La Valeur de la Science* [1905], which I take to be the most important of his books, the jewel in a quartet of precious stones.

My main reason for preferring this book is the primacy it gives to time ahead of space. My exposition of the philosophy of Poincaré will be strongly influenced by Giedymin [1982] who described the aim of Poincaré as follows: to examine the evolution of science and to show that progress is real, in spite of the radical changes transforming scientific theories.

Following Poincaré, *the search for truth* is the sole end worthy of science. Truth must be pursued in a spirit of righteousness, without prejudice and passion. But just as nature in itself is beautiful, so *the truth of nature is beautiful*, cf.p.6, Keats. If it were not, it would not be worth knowing, and life would not be worth living. Although there are worlds of difference between the passionate pursuit of beauty, the dispassionate search for truth, and the unselfish devotion to a higher purpose, these three cannot, and should not, be separated. As ideal values they are of the same kind, and whosoever truly loves the one cannot help loving the other two as well. The world is one. For this reason *art, science and morals* belong together.

Poincaré, being more dedicated to science, speaks primarily of *scientific truth*. In order to attain this goal, science must strive for *unity, simplicity*, and *objectivity*. Now experience is the only source of truth, and the ultimate arbiter of our theories, but it is we who decide how to search for truth, and when to trust the evidence; our hypotheses, our criteria and our method are our own choice and responsibility. Elaborating his own special brand of the widely accepted hypothetical deductive method, *Poincaré attempts to steer a middle course* between what he considers to be extremes: the pure *á-posteriori*, as propagated by the positivistic empiricism of his own time, and the pure *á-priori*, as advocated by the transcendental criticism of Kant.

Contrary to these extremes, he holds scientific theories to be constructions, or artefacts. The whole enterprise of natural science is *constructive*, aiming at a reconstruction of the formal relations which inhere in nature and, in this sense, science is also *descriptive*. Science never bothers about particular facts, its only concern being classes of facts; what it describes is the order or structure of facts, not their essence or substance: *disregarding substance and matter, science focusses on order and form*. Science seeks regularity in order to predict; only repeatable facts can be predicted. The only facts of relevance to science, therefore, are those that can be repeated, so the first step towards science is a preliminary classification of observables.

Scientific facts are nothing but common-sense facts expressed in the language of science. The language of exact science is an artificial one, namely the formal language of mathematics. The final outcome of scientific research is *a scientific theory*, and such theory is a harmonious *mathematical structure mapping the objective or invariant relations between observable facts.* Just as an artist selects those features of his model which perfect his picture of it, his refined sense for congruity induces him *á priori* to select precisely those facts which conform to his preconceived ideas and hypotheses of the universal harmony. But *it is a gross mistake to believe that the scientist creates his own facts: all that is manufactured in a fact is the formal language in which it is enunciated*, and it never depends on the scientist whether his prediction is fulfilled. Thus *empirical reality remains the ultimate arbiter of theoretical speculation.*

Poincaré thereby assumes a balanced position - apparently perfectly traditional - equally far away from all the excesses of nominalism or realism. It is customary to describe his position as *conventionalism*, although he did not make use of that term himself; but his particular version of conventionalism is moderate. Between the extremes, he discusses and repudiates the radical conventionalism, far from his own view, proposed by some contemporary devotees of idealism. I agree with Giedymin that the term *constructivism* may seem more appropriate.

In any case it is clear that his position assumes an empirical foundation amounting to the existence of a kind of *observational invariant* beneath all theoretical conventions. This invariant reality can be known up to the structural *isomorphism* of rival theories. Changes mostly concern ontologies and metaphors but seldom affect formal structures. This enables us to make steady progress in our knowledge of reality, but our knowledge remains limited in the sense that this *reality consists in no more than what is describable by a structure of group transformations*.

Both physics and geometry study invariants under the transformation of groups. Whereas *geometry* studies the properties of ideal space, *physics* studies the temporal changes of relations obtaining between objects situated in ideal spaces. Now *the passage of time is real* whereas *space is nothing but a word* wrongly supposed to refer to a real thing or frame: *real empty space simply does not exist*, and what experience informs us of is only the relations holding between solid bodies. However, the structure of time or space is not forced upon us by nature, it is we who impose it upon nature, though not by *á priori* intuition, but because it is convenient.

With respect to space we must distinguish between: *a*) solid *bodies* whose qualities are manifested to our bodily senses; *b*) their quantitative *relations* which are measurable relative to our standards fixed by convention; and eventually; *c*) the formal *spaces* of geometry proper. To Poincaré, *geometry* is just *the formal study of groups*. It is based on premisses chosen by considering their fruitfulness and appropriateness in our description of physical phenomena. As definitions, postulates, and rules of inference, its premisses lead to consequences which are derived by means of exact *analysis*. But the inventiveness of mathematical construction depends on imaginative intuition; for this reason *intuition* is our best guide to fruitful *synthesis,* cf. Kant. Geometrical space is taken to be continuous by convention and, a continuum having no intrinsic metric, the concept of metrical congruence is a convention. Thus, *on the whole, the assumptions of geometry are based on convention*, their foundation being neither synthetic, nor analytic.

Experiments teach us the relations of bodies to other bodies; they tell us nothing about the relations between bodies and space, nor about the relations between different parts of space. Following Poincaré, *the only relations existing in nature are the non-metrical relations of order* which are expressible in *topology*. Dismissing the Kantian position that the geometry of space is derived by synthesis *á priori*, Poincaré upheld a kind of *á-priorism* as regards the foundation of analysis and did not interpret the axioms of arithmetic as implicit definitions of primitives. Thus he did not extend his geometrical conventionalism to arithmetics which he took to depend on strict intuition *á priori*, based on whole numbers and the principle of mathematical induction. Further, he insisted that the consistency of geometry be evaluated relative to arithmetics.

Syntactically, *a geometry is nothing but an abstract formal language*. Formal languages may treat of various objects - points, lines, planes - yet they may be identical in their structure, due to group theoretical isomorphism. According to Poincaré it is natural to consider *geometries* as linguistic *frameworks* rather than as experimental conjectures; as frameworks they cannot be put to test, of course, but this does not imply that they must remain unchangeable. The choice of a geometry to correlate experiental facts is an opportunistic affair: does it provide us with the best means to solve the central problems of physics? To Poincaré, physical reality is knowable merely up to *the observational equivalence of alternative theoretical systems* and their structural isomorphisms. The question therefore is: can we avoid falsification of our physical ideas simply by constructing a new language which is formally translatable to the old one?

According to Giedymin, Poincaré adopted a general version of the Duhem-Quine thesis: *falsification is possible only relative to systems of hypotheses expressed in a fixed language.* Therefore, instead of blaming one or more of our hypotheses in face of contrary evidence, we may blame the experimental evidence, or we may avoid falsification by changing our language. The *language* of science is not fixed forever, but may be changed in response to experience.

Changing the *lexicon* is merely a subterfuge which tends to conceal the real problems. Changing the *syntax* of the language of science goes much deeper. It is not fruitful, however, to change our language in order to avoid falsification. Sometimes we have to accept facts as final and, if we do not, we condemn science to barrenness. But, on the whole, the language of science is based on conventional decision. Poincaré also extended his conventionalism to an analysis of the measurement of time and of the principles of physics. This was the reason for applying the term *conventionalism* not only to his epistemology but to his entire philosophy of science. Nevertheless, as already intimated, the term *constructivism* may cover his position better.

According to Poincaré, geometrical space is invented to ensure consistency regarding our reasoning about bodies and their relations: *spatial positions are not real properties of bodies, and bodies do not exist in real space, we just reason as if they did; further, temporal dates are not real properties of what happens to bodies*, but signifies the sum total of relations between events related to bodies. He finally exposed the *simultaneity* of instants and the congruence of durations to a trenchant analysis: distant simultaneity, e.g., is neither a datum of observation, nor is it a consequence of the temporal continuum, it just depends on convention. Therefore *time and space are both amorphous*, meaning that *they possess no real or intrinsic metric*.

Since the enunciation of physical laws varies with the conventions adopted, and since alternative conventions modify even the natural relations between the laws, it may be doubted whether there are among these laws any that can play the rôle of a *universal invariant* which is completely independent of linguistic conventions. However, if we introduce fictitious beings possessing senses analogous to ours, admitting the basic principles of our logic, it is a plausible conjecture that their language, however different to ours, will always be capable of translation into ours; but the possibility of translation implies the existence of something that is invariant. To translate between two different languages is precisely to disclose what remains invariant. The invariant laws, then, are those relating the supposed *ordinary facts*, whereas the relations between *scientific facts* always depend on certain basic conventions.

Poincaré distinguished between *three types of hypotheses:* formal principles, inductive generalisations and realistic interpretations. *A formal principle* is always a convention; as such it is *á priori* in a relative sense, its status being similar to that of a *real definition*; cf. Leibniz. *An inductive generalization* can be viewed as an *experiental law*, thus it is subject to repeated revision, but may at a later stage be promoted to the status of principle; such a law expresses a relation between two terms, a conceptual one and a factual one. If a law is elevated to the status of principle, a third term is introduced to mediate between the first two; hence the first relation is split up into two other relations, viz., a theoretical one between two conceptual terms, and an empirical one between a conceptual and a factual term. *A realistic interpretation* of the terms is neutral if it does not affect the relations between the terms, although the terms may be changed; the same geometry, e.g., may result whether we begin with points, with lines, or with planes.

The physics of our own time is the physics of the principles, said Poincaré. Any law can be broken up into an *á priori* principle and an *á posteriori* law; for this reason the number of scientific principles has increased and still increases while a conceptual structure is taking form. But however far the partition be pushed, there will always remain laws which are in need of being tested against experience and, if not, science as we know it would be brought to an end. We cannot satisfy all conceivable principles at once in the face of evidence, and if a principle ceases to be fertile, experiment will have condemned it without contradicting it; the reason is that *if a principle cannot be refuted by any experience, it is no longer informative:* we can infer nothing from it; and, of course, it is *useless to heap up hypotheses*.

In todays physics empirical generalizations are steadily upgraded to theoretical principles. From an epistemological point of view, the difference between geometry and physics is that, whereas all *principles of geometry* are conventional, only some of the *principles of physics* are. The importance of principles in current physics seems to be growing whereas our ability to find experimental results enabling us to discriminate between theories appears to be diminishing.

It is interesting to compare this stance of Poincaré to the position of Niels Bohr, that the description of an experimental apparatus must always be given in classical terms, even in quantum mechanics. Although the precise borderline between object and subject, or between reality and apparatus, seems to be made by an arbitrary decision, a cut must certainly be made. But, in contrast to Bohr, Poincaré remained open to a revision of classical physics.

As regards the contribution of Poincaré to Special Relativity ***(SR)***, my own position is in line with the judgment of Keswani [1964f.]: *Poincaré had the whole theory, including 'ondes gravifique', and had it before Einstein, the only important difference being one of emphasis*. The fundamental questions presented by the *Relativité Resteinte*, as Poincaré saw it, was this: will not the principle of relativity, as stated by Lorentz, impose upon us entirely new notions of time and space, thereby forcing us to give up our most cherished classical ideas?

But how could Poincaré describe the principle of relativity as being unfalsifiable and yet think of giving it up in face of seemingly negative experimental results? There is a very straight-forward answer to this question: on a par with Newton, he did not want to heap up hypotheses! However, he did not accept that ***SR***, or ***GR***, could ever force us to renounce conventionalism: even spacetime is a conventional framework that should not be confused with external reality. He saw space as amorphous, so he dismissed the intrinsic metric proposed by Einstein.

Poincaré was well acquainted with the work of Einstein, but did not credit him with the invention of ***SR***. After emphasizing that the simultaneity of two events, as well as the equality of two durations, should be so defined that the natural laws may become as simple as possible, he credited Lorentz for having saved relativity by means of his ingenious idea of local time. Lorentz later, paying tribute to Poincaré for his great contributions to physics, praised him for having stated the relativistic transformations in their most convenient form, ahead of Einstein and Minkowski. Lorentz had claimed that the forces of physics should be defined so as to make them invariant to his transformations. Poincaré, accepting this claim, tried to modify Newton's law of gravitation by constructing a Lorentz-invariant action-at-a-distance theory. In the view of North [1965]: had the sympathy not been so decisively in favour of a field theory of gravity, "Poincaré's memoirs might have become a turning point in the history of the subject".

It has been noticed by Stump [1989] that the burden of conventionalism is to explain the conventionality of the fundamental principles of science in a relationist way, without relying on arguments of under-determination. So, in order to be consistent, conventionalism must explain the relational origin of both gravitation and inertia. In fact, Poincaré believed that acceleration depends only on the external relations between bodies: velocity and acceleration being on a par, both have to be relative. As he felt obliged to find a solution in terms of bodies and the forces acting upon them, he did not consider the possibility of reducing gravity to spacetime structure. According to Stump, Einstein's spacetime theory of gravitation constitutes a disproof of the possibility of a pure relationalist framework. But this view is premature, to say the least.

Thus Roxburgh & Tavakol [1975] have written an important paper displaying the close affinity between the gravitational theories of Poincaré and Milne. They see the great value of Poincaré's action-at-a-distance theory in the fact that it has led to the discovery of an entire family of consistent theories which cannot be geometrized in a Riemannian manifold, but only in the more complex framework of Finsler. The cosmological solutions for these theories are derivable by means of a generalized version of the kinematic technique invented by Milne.

3. ARTHUR STANLEY EDDINGTON (1882-1944)

It is right to say that Kant anticipated to a remarkable extent the ideas to which we are now being impelled by the modern development of physics. Eddington [1939]

The scientific career of Eddington was very extraordinary. He took many degrees and was appointed Plumian professor of astronomy at Cambridge University when he was only 31. He made great contributions to astrophysics for which he was deservedly famous, and he wrote a lot of books, scientific and popular, which were much acclaimed. At his heigth he enjoyed a public authority almost second to none; but the humorous style of his books, exceedingly well written, was also provocative and earned him much opposition. He exposed himself and became a favourite target of criticism for positivist philosophers armed with heavy irony, but with scant sense of humour. What specifically arose the hostile feelings of many of his scientific collegues was his insistence on the possibility of mapping the structure of the universe *á priori*.

Whittaker [1947] ascribed the following principle to Eddington: *It is possible to calculate the exact values of all pure numbers describing timeless relations between the basic constants of nature by á priori mathematical deduction from epistemological principles.*

By scientific knowledge *á priori* Eddington understood knowledge that is prior to actual measurement, but not prior to exact specification of the operational procedures of measurement. He claimed to have expressed in mathematical symbols what the physicist thinks he is doing when he is measuring things. Whittaker [ibid.] described Eddington as a modern Archimedes. Archimedes, by calculating π, the ratio of the area of a circle to its squared radius, assumed qualitative geometry in order to deduce quantitative geometry. Eddington allowed himself to use everything physical except the numerical values of constants of nature: these he claimed himself able to deduce from epistemological principles, in analogy to the deduction of π.

According to Galileo, the goal of natural science is to measure what can be measured and to make measurable what cannot yet be measured. Science focusses on the quantitative aspect of nature by effecting a reduction of quality to quantity. To say that science is based on experiment and observation performed by means of apparatus is to say that it is based on counting and on the readings of instruments. Eddington called such readings *pointer-readings,* the primary ones giving the intensities of the qualitities measured, and the secondary ones giving their spacetime location, the context of the experiment being described by pointer-readings of a tertiary order.

Pointer-readings mark the coincidence of spacetime events and are quantities produced by our operations; they are not properties of nature, but relations between object and observer. All the observable variety in the universe stems from the diversity of relations between entities; nothing in the world forces us to split it up into identical units, this is just our way of thinking. But, if we consider the intrinsic nature of the entities related, nothing is left but sameness. Eddington therefore said that there is only one kind of fundamental particle: all the rich variety of elementary particles is just a manifestation in disguise of this particular sort!

Now it is possible for a group of sensations in a mind to have the same structure as a group of sensations in another mind. It is also possible for groups of entities to display the same structure, although their true nature and their properties remain unknown to us. So we can get structural knowledge of "things outside ourselves". The recognition that all physical knowledge is structural *makes obsolete the dualism between mind and matter*, Eddington claimed.

In physics, a variety of observations can be termed structural. It was Eddington's aim to construct their fundamental theory. The result of such theory is an extensive unification between the different branches of physics. Eddington did not search for new laws, he wanted to explain those already known. It is the invariance, under different circumstances, of elementary particles with simple attributes that provides us with the ultimate numerical standards of nature.

The pure numbers of nature arise as ratios of the numbers of dimensions of certain phase spaces, and our task is to calculate the number of dimensions of such spaces, he insisted.

To solve this task Eddington invented his famous calculus of *E*-numbers, a generalized version of the even more famous Hamiltonian algebra of *quaternions*. Hamilton interpreted his quaternion algebra in Kantian terms as "the science of pure time": *arithmetic* maps the structure of our intuition of *pure time* whereas *geometry* maps the structure of our intuition of *pure space*, said Kant. Eddington, by analogy, saw his own *E*-algebra as "the science of spacetime".

According to Yolton [1960], the modern edifice of natural science is developed so far that most of the relevant data in many fields have already been collected, and so the remaining task is to unify them, formulating them in a deductive system. Yolton opines that Eddington in fact made no real *á priori* deduction of the constants of nature. The codes of empirical science are not violated by focussing upon its theoretical aspects. The laws determining the quantitative results of observation are sometimes inferable from our operational specification of the relevant observational procedures. Basic hypotheses can often be replaced by epistemological principles. New data emerging, it is often found that a different system is required. Temporary set-backs, however, cannot block the general trend of science towards unification.

Eddington has attracted quite a number of followers to join his search for a deductive explanation of the strange numerical coincidences of the universe: Paul Dirac, Pascual Jordan, Erwin Schrödinger, Hermann Bondi, Peter Landsberg, and Peter Rowlands. A small society, *ANPA*, standing for *Alternative Natural Philosophy Association*, has been formed by scientists devoted to the quest for explaining these numbers, and Eddington has recently been conferred a posthumous honorary membership. Ted Bastin & Clive Kilmister, leading members of *ANPA*, have written a series of papers on Eddington's *Fundamental Theory*, cf. Kilmister [1966], which has been followed by a study of Kilmister & Tupper [1962] on *Eddington's Statistical Theory.* Assuming the quantitative aspects of the universe to be finite and discrete, David Mc Goveran of *ANPA* has used binary algebra and computer theory to improve the combinatorial hierarchy of F. Parker-Rhodes, now accepted by *ANPA* as the common ground for further research.

4. EDWARD ARTHUR MILNE (1896-1950)

The so-called principle of induction .. has no content ... It is a piece of out-moded furniture, and in fundamental investigations it had better be scrapped.

There is no entity 'physical space'; there is only the abstract space chosen by the physicist as a structure in which to plot phenomena, some choices giving simpler theorems than others.

The essence of scientific freedom is the right to come to conclusions which differ from those of the majority. Milne [1951]

As a scientist Milne never attained the fame or prominence of Eddington, nor did he become victim of so bitter and fierce an opposition; but that in itself does not make him less interesting, nor less important. His feat as a cosmologist was to construct an exceedingly simple model of a universe implying the uniform dispersion of its contents of galaxies, in accordance with a cosmological principle demanding a specific type of cosmic symmetry (isotropy), of a substratum of fundamental particles. He also showed how the superposition of a secondary set of freely moving accidental particles on this substratum gives rise to something like gravity.

The main idea of Milne was that the laws of nature can be deduced rationally by taking an individual observer's awareness of a temporal sequence of events as one's point of departure. His central hypothesis, that the laws of nature are akin to geometrical theorems, places him on a par with Eddington. But Milne did not depend on Eddington. His claim, that it is possible to deduce the inverse square law of gravitation, together with the sign of gravitation, *á priori* from some excedingly simple premisses, thereby reducing gravity to inertia, may sound shocking to most scientists. What arguments did he adduce in support of this startling view?

At the dawn of history, the theorems of geometry were regarded as principles of nature. All we know of Egyptian mathematics, at least, is consistent with the view that the Egyptians regarded regularities like those summed up in the Pythagorean proposition as laws of nature. These empirical regularities were discovered by drawing up different triangles, measuring them, and experimenting with them; but as "observational laws" they were nothing but brute facts. The Greeks, deducing the laws from combinations of simpler statements, postulates, or axioms, later transformed geometry into an exact science wherein everything follows from pure theory. They thereby showed the possibility of eliminating brute facts from science.

In modern presentations of geometry, the axioms are neither brute facts nor empirical statements; instead they are definitions, i.e., minimal descriptions of what we are talking about. The theorems derived from the axioms are valid when the process of inference contains no flaw, so their truth does not depend on verification. *The tendency of all exact sciences is to pass from the Egyptian inductive phase to the Greek deductive phase*, the only question being how far this can be carried out. The extent to which it can be carried out is at the same time our measure of the degree to which we can regard the universe as being rational, Milne claimed.

The laws of geometry are derivable by pure deduction, this is evident to all; so why not assume that the laws of dynamics are likewise derivable by pure deduction? Whether rigorous deduction is possible is a question that cannot be decided *á priori.* One cannot begin by stating a programme of this kind, and then just carry it out, it derives from the *á posteriori* experience of pushing deduction as far as possible. When we introduce operational definitions, a sufficient description of the relations between real entities is provided; appeal to brute fact is unnecessary. But experience is needed is to test whether a specific world model stands up to fact.

Milne's technique of radar-signals was refined by Walker [1935f.] and Whitrow [1961], as well as by Törnebohm [1957&1963] and Schutz [1973]. A popular version of their ideas is found in the famous *k*-calculus of Bondi. Lucas [1973], referring to Whitrow & Milne, approves "their transcendental derivation of the Lorentz transformations" as being probably the best of all possible ways in which "bewindowed Leibnizian monads" can recover their lost harmony. According to North [1965], the independent derivations by Robertson and Walker of the ***RWM*** standard metric for cosmology are based on assumptions inspired by Milne.

Together with McCrea, Milne intended to revive the classical cosmology of Newton in a climate completely dominated by the ideas of Einstein, cf. North. How far this development can be taken has since been shown by Landsberg & Evans [1977]. These Newtonian world-models, which were never intented to stand up to observational test, should be carefully distinguished from the world-model of Milne as presented in his monographs of [1935 &1951].

More relevant to our present purpose, however, are the attempts of Walker [1940, 1943] and Schutz [1973] to transform the kinematic ideas of ***SR*** & ***KR*** into an exact deductive science. Their studies have in a convincing manner disclosed the unique significance of the method of radar-signals as a means to enlighten problems of modern relativity theory.

Nevertheless, it is a question if the results obtained by Walker and Schutz have benefitted sufficiently from the ingenious constructivist ideas of Poincaré. What they have obtained is a mapping of the topology of current relativity theory, and what remains to be done is to expose the conventionality inherent in the metric. This reflects, for instance, on the standard definition of simultaneity at a distance. Personally, I believe Poincaré would have welcomed an attempt to show that Einstein's dissolution of distant simultaneity is not an inevitable consequence of the topology of ***SR***, but depends on his particular (conventional) choice of spacetime metric.

In cosmology, we push the deductive aspect as far as possible. But it seems as if Milne was able to make his mathematics yield more than he had put into it: his output seemed to exceed his input. If this is right, then it is no longer true that only synthetic propositions contain new knowledge; analytic propositions may likewise do that when they add to their premisses "the leaven of the deductive process" (Milne). If we accept that the universe is rational, the hope of many scientists that the constants of nature can somehow be deduced may not be in vain.

Any hard-baked, or hard-boiled, scientist will traditionally hold that science and religion, whilst "on nodding terms" (Milne), have no immediate bearing on one another. On the contrary Milne held that one cannot study cosmology unless one has a "religious attitude" to the universe. Einstein, in fact, meant something similar. Cosmology assumes the rationality of the universe, but is unable to give any reason for it except the cause of nature being a rational Creator.

To Milne, the creation of the universe remains the sôle and ultimate irrationality.

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