Mogens True Wegener

NEW AXIOMS FOR COSMOLOGY

FROM SPACE-TIME VIA TIME-SPACE TO SUPER-TIME

Revised version (2015) of a paper presented at the PIRT Conference
'Mathematics, Physics and Philosophy in the Interpretations of Relativity Theory'
Loránd Eötvös University, Budapest 2009.

=//=

Summary:

In the present paper it is shown how it is possible by means of a time-based concept of equidistance to construct a spatial geometry for relativistic cosmology. In analogy to a sphere defined as the geometrical site for all those points which are equidistant from a given point, we construct a plane as the site for all those points that are equidistant from two points, and a line as the site for all those points equidistant from three points.

Having defined parallellity and perpendicularity we proceed to define the cosmic substrate in a way analogous to the cosmological principle of the Cusan. Assuming that we can always construe the center point for any three non-collinear members of this substrate, we can prove simultaneity to be universally transitive for all members of the substrate if the simultaneity be defined indirectly by means of equidistance.

=//=

PREAMBLE

An axiomatization of relativitic cosmology may be construed with various aims in mind. One goal is to codify pet ideas and entrenched theories thus giving boost to scientific dogma. Quite another is to invent a model depicting some basic traits of the universe in order to test that structure against experience. It is the second purpose that has motivated this article.

Instead of anticipating what is written in my paper, I prefer to say a little more about why I find it important to put focus on such a very general structure displaying such particular traits. Like the philosopher Bergson, I always felt that the so-called "spatialization of time" is a great mistake on behalf of science, indeed the ultimate catastrophe. The idea of a "block universe" wherein nothing happens, everything existing of eternity though in a timeless way, is foreign to experience, and the phantasy of "time-warps" and "space-time tunnels" leading to other worlds is, in my opinion, nothing but idle speculation bolstered up with subtle mathematics.

Eddington once said: "In physics everything depends on the insight with which the ideas are handled *before* they reach the mathematical stage". These words are true, wise and pertinent; but, like Eddington, most physicists still turn to Einstein as if *his* ideas were the highest wisdom. Now, according to Einstein, time is what is read off our clocks and, if our clocks are retarded, time itself must be dilated (this inference was made by Minkowski, then adopted by Einstein). Further Maxwell's equations involve a constant c that is naturally interpreted as a limiting speed, viz., that of light. What is more natural, then, than to insist that time is another form of space?

Long forgotten is the doctrine of Descartes, that mind and matter are two different kinds of "being" with one property in common, viz. their temporal duration or "durée", cf. Bergson. Equally eclipsed by oblivion is the insight of Kant that, whereas the intuition of *space* yields the *geometric* form of all *external* experience, the intuition of *time* yields the *arithmetic* form not only of internal experience, but of all experience, *external as well as internal*. It was precisely this insight which inspired Hamilton to invent his quaternions, so fruitful to physics.

But is it not "a reactionary move" to propose an axiomatics to revitalize those outmoded concepts of absolute simultaneity and universal time so characteristic of classical physics? Well, I have not the slightest doubt that professor Nemeti and his followers will consider it that way. As far as I understand, they want to base their axiomatics on the socalled light-cone geometry. The point to be noticed here is that the opening angle of the cone to its axis is representative for the velocity of light: for the one-way velocity of light, that is.

Now the standard version of the special theory of relativity ***(SR)*** is distinguishable from other competing theories not only by its two basic principles, that of relativity and that of the constancy of the velocity of light, but similarly by the fact that its definition of simultaneity at a distance is determined purely by convention. This, of course, entails that its definition of the one-way velocity of light must be conventional too! That this is so was shown by John Winnie [1970], and in another way by my friend Peter Øhrstrøm [2000].

SR *is obviously equivalent to a host of theories, each with its own one-way light-speed. So, according to* ***SR****, whereas the average or two-way velocity of light is a universal constant, the one-way velocity of light - hence the opening angle of its light cones - is wholly arbitrary. This, as far as I see it, puts the geometric light-cone enterprise completely in jeopardy.*

Mogens True Wegener

To this argument it might be objected that, recalling Reichenbach's ϵ-constant, Einstein's choice of $\epsilon = \frac{1}{2}$ is unique in the sense that it accords with the value obtained by infinitely slow clock transport. It may further be adduced that Malament has presented a famous argument to the effect that the value $\epsilon = \frac{1}{2}$ is the only one that allows for a reversal of the direction of time. Finally, this value appears to be indispensable to the standard definition of a reference frame.

But these objections are nothing but subterfuges. Just like the definition of simultaneity at a distance, the one-way velocity of light for a certain distance is either conventional, or it is not. If we stick to ***SR*** it is certainly conventional, that is, indifferent to experiment and observation, and then the standard concept of a reference frame is nothing but a convenient fiction, or a fake. If it can be shown that it is not conventional, we shall have to search for an alternative theory, involving a different concept of reference frame than the usual one of standard relativity.

It is commonplace that physicists - instead of taking incompatibilities and contradictions for what they are, namely, incompatibilities and contradictions - try by all means to evade them by making distinctions so subtle that no one can sort them out. This applies in particular to the attempts at unifying ***GR*** with ***QM***. Very few seem willing to face the fact that ***GR*** and ***QM*** are incompatible and that all attempts to unify them inevitably lead us into a mess of contradictions.

Now dr. Rowlands, in his conference paper, suggests that gravity may be instantaneous. This, in my opinion, is a suggestion that might well be true and should not be lightly dismissed. He admits that "there is, therefore, an incompatibility between the space-time structure of our observations (supposing it to be that of standard ***SR***) and the space-time structure we require to set up our gravitational equations" but, adds he, "this is not an uncommon occurrence in physics and there is a well known solution". To a logician such words sounds a little distressing.

I agree with dr. Rowlands that "quantum gravity is a meaningless idea". But I believe the same holds of his own proposal that, if we apply a Lorentzian space-time to a system subject to non-local gravitation, this incompatibility can be circumvented by introducing something called "fictitious effects" to compensate for an otherwise outright contradiction. It is clear that in order to vindicate the view that gravitational effects are instantaneous, the notion of simultaneity at a distance must be uniquely definable, and this it is not with standard ***SR***. Here logic holds sway, and I see no reason to endorse current attempts to modify basic logic in agreement with physics. Nevertheless, dr. Rowland's *distinction between instantaneous and observational structure* is certainly relevant, compare the distinction between *World-Map* and *World-View*, ch.5 §7.

Another of my friends, professor Selleri, points to what I think may be a better solution. Having in his forthcoming new book on *Weak Relativity* assembled and discussed a number of observations and experiments in support of the revolutionary idea that simultaneity, after all, is absolute, he proceeds to develop some new inertial transformations to accomodate that idea. Accepting the evidence here accumulated, I agree that we shall have to search for a new theory. An essential characteristic of the inertial transformations proposed and generalized in his book is that the dependence of the temporal coordinate on the longitudinal spatial one so distinctive of standard ***SR*** is here suspended, so that only the well known γ-factor of ***SR*** remains relevant.

The same remark applies to another non-standard theory developed by my friend Tom Phipps, who in his monumental book *Heretical Verities* [1986] has constructed a full-fledged socalled neo-Hertzian electrodynamics in due respect to almost all current relativistic evidence. Regrettably, however, these two heretic spirits do not agree concerning the spatial coordinate.

Furthermore, they are both reluctant to accept the full validity of the strong relativity principle. In my own opinion this is a very serious drawback of their respective non-standard theories and if I shared their view, I might not have been that eager to invite our guest professor Ungar.

In any case I agree with professor Ungar that the standard Lorentz transformations should be applicable to all equivalent observers in a universe which is in a state of uniform expansion. The latter condition, of course, imposes a problematic restriction on our cosmological theories; so we shall have to search for other transformations if the expansion, as it seems, is accelerating. but the need for new transformations is also urgent if the observers involved are not equivalent. Now the statement that the expansion is accelerating must refer to an accepted standard, and the usual standard is given by the size of our own body, i.e., by the size of its constituting atoms.

This standard being extrapolated *ad infinitum* in space we get our usual reference frames, and if such frames are synchronized by the usual convention, our choice of an origo seems free. What I surmise is that this is a mistake involving the dissolution of classical simultaneity; so if, with Selleri and Phipps, we want to keep simultaneity absolute, this mistake should be avoided. I agree that all this may sound strange. How can I both accept a weak concept of relativity for the reason that it allows simultaneity to be absolute and yet endorse a strong relativity principle? An explanation is obviously urgent. But my point is that a distinction must be made.

A way out of the impasse was shown long ago by E.A. Milne, in his *Kinematic Relativity*. In this classical monograph he proposed a world-model whose structure is determined by an infinite substratum of perfectly equivalent fundamental particles exposed to uniform dispersion. This substratum, the members of which are subject to the *strong relativity* principle, was then supposed to be covered by a layer of accidental particles subject to a weaker kind of relativity. Milne's universe thus consists of two sets of particles: the substratum of *fundamental particles* subject to *strong relativity* as formulated in the principle of cosmic isotropy, and a further layer of *accidental particles* breaking the cosmic symmetry, thus only subject to *weak relativity*.

Milne denounced the Friedmann equations of ***GR***, hence his model is not Friedmannian. Instead of assuming gravitation, hiding it in the curvature of space, it was his aim to deduce it. He began by deriving the Lorentz transformations and showing them to hold good between the comoving frames of all fundamental observers, *implying that each frame has its natural origo in the only fundamental observer at rest in that frame*, all others receding with uniform velocity. His assistant Walker then showed the standard coordinates of ***SR*** to be transmutable into some other coordinates mapping the universe to be in a state of uniform expansion in cosmic time.

As I see it, the difference between the many *private* Lorentzian 3-spaces, each frame correlated with its own Einsteinian t-time, and the single, unique and *public* Robertson-Walker 3-space correlated with the cosmic T-time of Walker, is also the key to an understanding of the ingenious non-commutative non-associative algebra invented and studied by professor Ungar.

The Milne model, which is very clean, was later adopted by Törnebohm and Prokhovnik; it can be described as having its origin not in a "big bang", but in a transcendent singularity. Applying his cosmological principle to the Lorentz transformations, and making an important distinction between *world map* (the universe as it is in itself *at an instant of cosmic time*) and *world view* (the universe as it appears to an observer, *sliced up in different temporal layers*), cf. the famous one of Kant between "das Ding an sich" and "das Ding für uns", Milne was able to describe precisely how his world model of uniform dispersion relates to observation.

How did this remarkable feat fall into oblivion, so that Milne and his kinematic relativity theory is hardly mentioned in more recent expositions of cosmology? Well, one explanation is that the fame of Einstein has reached mythological dimensions overshadowing that of all others. In our modern culture, what is greater than to become depicted on the side of a shopping bag? Every child knows the name of Einstein, but who cares about that of Plato, except philosophers? Another explanation is that Milne's gravitational theory did not stand up to observational test, but was surpassed and excelled in that respect by Einstein's. But this is just another myth.

For the sake of fairness one must allow a theory to become developed and, as reported by Whittaker in his *History of the Theories of Aether and Electricity vol.2*, Walker early suggested a variational principle based on ***SR*** that almost reproduced the observational results of ***GR***, so he issued a much ignored warning: don't accept a theory with only scant experiental support. It later became almost a sport to construe formulae based on ***SR*** and imitating the results of ***GR***. I have given two myself, and the issue has been studied in depth by Rowlands [1994, 2007].

I only want to urge the ingenuity of Milne's basic idea: to derive gravity from asymmetry. We have recently learned much about broken symmetries leading to differentiation of forces: strong, weak, electromagnetic, etc., but hitherto noone has succeeded to incorporate gravitation. The project of quantum gravity is still unsolved, despite the endless efforts of mathematicians. But a solution may be close at hand and much simpler than expected: just follow Milne!

According to Milne's kinematic relativity theory, the substratum is an ideal, infinite and dense set of equivalent particles subject to hydrodynamic continuity and obeying Hubble's law. Since he assumed the universe to be in a state of *uniform dispersion* - expressed with the metric of Walker: in a state of *uniform expansion* - his universe embodies a perfect *compass of inertia*, to use a felicitous phrase of Weyl. This means that it is impossible to construe an inertial frame which is not in the end identifiable with the frame comoving with some fundamental observer.

But, as already said, *only one fundamental observer can be at rest in an inertial frame: this observer constitutes the natural origo of that frame*, so we are not free to choose any other. The substratum, as told, is covered by a layer of accidental particles. Now, in contrast to the fundamental particles which are at rest, each one in its own comoving frame, the position and velocity of an accidental particle is completely described by reference to two fundamental ones: that with which it is momentarily coincides and that relative to which it is momentarily at rest, and the closer these two are to each other, the more the particle tends to fundamental status.

Milne's model, like all other models of the universe, is subject to conservation of energy. Now an accidental particle, due to its velocity, possesses a certain amount of kinetic energy as estimated by that fundamental observer with which it momentarily coincides. But fundamental observers are equivalent. The accidental particle therefore possesses exactly the same amount of energy as estimated by any other fundamental observer, and only its form may seem different. How, in particular, must its energy be perceived by that observer relative to which it is at rest? Only one possibility is left open to him: the energy must appear as being potential, or dynamic. *Thus, what to the first observer appears as* ***inertia,*** *to the second one must appear as* ***gravity***! With Milne, cosmic time is conditioned by symmetry, gravity is caused by asymmetry.

I agree with professor Ungar that there is an intimate connexion between special relativity and hyperbolic geometry. The same does dr. Barrett, and so did Milne. But there is a difference. Whereas professor Ungar ascribes a hyperbolic character to the *velocity space* of ***SR***, dr. Barrett also tends to view its *position space* as being hyperbolic. Milne, by contrast, speaks of a choice.

*I earlier sided with dr. Barrett, but at this point I have changed my mind; see pp.11&72. I now accept the standard light-invariance of **SR** but renounce instead on the **RWM**.*

=//=

1. INTRODUCTION.

The Greeks prescribed that proofs of geometry should be made solely by means of a ruler and a pair of compasses. In this paper it will be shown how the ruler can be dispensed with, and how all concepts of geometry can be constructed by means of "a temporal pair of compasses", in a way reminiscent of, but also different from, the method devised by Georg Mohr [1672].

Geometry treats of spatial structure and, according to Descartes, space is real, because extension is a sign of substance on a par with cogitation. But Leibniz denied this, arguing that extension, being infinitely divisible, cannot be substantial. According to Leibniz, space is not real, but neither is it illusive; rather it is *well-founded appearance*. To this view he was inspired by Plato who, in his *Timaios*, held that space is neither pure concept (*idéa*), nor pure appearance (*fainómenon*), but "something in between" so that, seeing it "as in a dream", we cannot tell what it is, but feel that it has to be, in order for events to take place. Thus, to Plato, space is necessary in order to give room for events, whereas time alone is "an image in motion of eternity".

Contrary to common prejudice the Platonic view of space is supported by modern science Thus, in their excellent monograph *On General Relativity*, Mercier, Treder & Yourgrau [1979] unanimously insist that "*there is no such thing as real space*" (p.134). Further Mercier [2000], in a more philosophical article, advocates the view that '*space-time*' should be reconstructed as '*time-space*', or '*super-time*'. But, more than half a century before this, Milne already proposed the same idea, first in several papers, partly written in collaboration with his student Whitrow, then in his own ingenious *Kinematic Relativity* [1948/1951], wherein he constructed an entirely new cosmology from first terms based on time as its fundamental concept. This cosmology was described by Merleau-Ponty [1965] as "*a Leibnizian monadology*" translated into mathematics. The present paper may be read as a modest attempt to elaborate on the very same idea.

According to Whitrow [1972], "Leibniz's principle (of a pre-established harmony, MTW) is equivalent to the postulate of a single universal time; we must therefore discard this principle - i.e., if we are to reconcile Leibniz's way of regarding time with Einstein's theory of relativity". But why attempt to reconcile Leibniz with Einstein? As shown by Walker - another student of Milne's who, independently of Robertson, invented the relativistic standard metric of universes that are everywhere isotropic - such universes not only allow, but outright demand, a universal time parameter determining the spatial scale factor and, as demonstrated by Törnebohm [1963], even special relativity gives place to two distinct concepts of time and two different definitions of simultaneity: one relative, another absolute. Therefore, as argued by myself, Wegener [2004], we have every reason to search for a new formalization of special relativity consistent with the idea of a Cosmic Time, even if this goes against "the spirit of Einstein".

=//=

2. FORMAL PRESENTATION (2015)

In what follows I have found it more important to convey an impression of the main ideas rather than to offer an impeccable presentation of some full-fledged formalization.

The axiomatics is sketched by means of 1st order predicate logic (***FOL***) or quantification theory (***QT***) supplemented with basic set theory, $\forall$ being the universal quantor, $\exists$ the existential quantor, $\Rightarrow$, $\wedge$, $\vee$, $\neg$, $\Leftrightarrow$ being the constants of implication, conjunction, disjunction, negation, and equivalence, and $=$ and $\simeq$ being the relations of identity and similarity. $\forall x : Ax \Rightarrow Bx$, $\exists x : Ax \wedge Bx$ are well-formed formulae, and $\{.,x,y,z,.\}$, $\{x\,|\,(\textit{description})\}$ are simple sets. Pauses are marked by points. Quantors are omitted from definitions. Definitions are indicated by $\equiv$ which, depending on the context, may be read either as $=_{df}$ (i.e. identity: "*is/are*") or as $\Leftrightarrow_{df}$ (i.e. equivalence: "*iff*"). Finally, $\rightarrow$ denotes a signal, or the exchange of a single "photon" (a one-one relation between an emission at one particle and an absorption at another particle), while $\preceq$ betokens the causal relation of precedence, or succession (of signals).

Df.1: *The Universe.*

$\boldsymbol{Universe} \;.\equiv.\; \left\{ \mathrm{P}^i \,|\, \mathrm{P}^i \equiv \{..,P^i_1,P^i_2,P^i_3,..\} \right\}$

Translation: The universe is the set of all existing *points*, or *particles* - or *particle-observers*, or *observer-particles*, or ***monads*** - which themselves are (consist of) sets of observational events.

AX.1 *Causation by Signals.*

$\forall \mathrm{P}, \forall \mathrm{P}', \forall P_a : \; P_a \in \mathrm{P} \;.\Rightarrow.\; \exists 1\, P'_e : \; P'_e \in \mathrm{P}' \wedge P'_e \rightarrow P_a$

Translation: For any two particles $\mathrm{P}\,\&\,\mathrm{P}'$, and for any event P_a in P (absorption of a "photon"), there was one, and only one, event P'_e in P' (emission of a "photon") which was the cause of P_a.
Comment: One must distinguish *backwards inference*, i.e. the ***explanation*** of a *present effect* by its *past cause*, from *forwards inference*, i.e. the ***prediction*** of a *future effect* by its *present cause*. In order to avoid discussing probabilities we put focus on explanation rather than prediction. Ax.1 says that all events, e.g. state P_a of P, were caused by preceding states of other particles. So *there was no first event* - any possible first event was *beyond all possible experience*.

Df.2 *Successive Causation.*

$P_e \preceq P'_a \;.\equiv.\; \exists \mathrm{P}, \exists \mathrm{P}' : P_e \in \mathrm{P} \wedge P'_a \in \mathrm{P}' \wedge P_e \rightarrow \ldots \rightarrow P'_a$

Translation: For any pair of events, P_e in P and P'_a in P', we shall say that the event of absorption P'_a *was* caused (not: *will be* caused) by the event of emission P_e, iff a signal was emitted by P at event P_e and, possibly after transmission by other events, observed by P' at the event P'_a.

AX.2 *Transitivity of Succession.*

$P'_1 \preceq P''_2 \wedge P''_2 \preceq P'''_3 \;.\Rightarrow.\; P'_1 \preceq P'''_3$

Translation: If P'_1 caused P''_2 and if P''_2 caused P'''_3, then P'_1 caused P'''_3.

Df.3 *Momentary Coincidence.*

$\mathrm{PP}'(P'_r) = 0 \;.\equiv.\; \exists P_e, \exists P_a : \; P_e, P_a \in \mathrm{P} \wedge P_e \preceq P'_r \preceq P_a \wedge P_e = P_a$

Translation: The ("light-time") distance between P and P$'$ at event P'_r in P$'$ was zero iff there were events P_e and P_a in P, so that P_e caused P'_r which caused P_a, but P_e coincided with P_a.

Df.4 *Permanent Coincidence.*

$\mathrm{P} = \mathrm{P}' \,.\equiv.\, \forall P'_r : \mathrm{PP}'(P'_r) = 0$

Translation: Two particles, P & P$'$, are *identical* iff their momentary distance is always zero.

Df.5 *Momentary Equidistance.*

$\mathrm{PP}'(P'_r) = \mathrm{PP}''(P''_r) \,.\equiv.\, \exists P_e, \exists P_a : P_e, P_a \in \mathrm{P} \wedge P_e \to P'_r \to P_a \wedge P_e \to P''_r \to P_a$

Translation: The light-time distance from P to P$'$ at the event P'_r in P$'$ equalled the light-time distance from P to P$''$ at the event P''_r in P$''$ iff both events of reflection were caused by the same event of emission P_e in P and coincidentially caused the same event of absorption P_a in P.
Comment: This temporal definition of ***equidistance*** *is basic to our axiomatization of geometry*; being comparable to the span of "a pair of compasses", it justifies our use of that metaphor. The notion of equidistance presupposes that "photons" are transmitted with invariant "two-way" speed in all directions, i.e. *isotropically*. We do *not* assume a constant "one way light speed".

Df.6 *Permanent Equidistance.*

$\mathrm{PP}' = \mathrm{PP}'' \,.\equiv.\, \forall P'_r, \forall P''_r : \mathrm{PP}'(P'_r) = \mathrm{PP}''(P''_r)$

Translation: The distances from P to P$'$ & P" remain equal iff they are equal at all events.

Df.7 *Momentarily Smaller/Greater Distance.*

$\mathrm{PP}'(P'_r) < \mathrm{PP}''(P''_r) \,.\equiv.$

$\exists P_e, \exists P_a, \exists P_{a'} : P_e, P_a, P_{a'} \in \mathrm{P} \wedge P_e \to P'_r \to P_a \wedge P_e \to P''_r \to P_{a'}$

$\wedge\ \exists \mathrm{P}''', \exists P'''_r : \mathrm{PP}'''(P'''_r) \neq 0 \wedge P_a \to P'''_r \to P_{a'}$

Translation: Distance PP$'$ at P'_r in P$'$ was *smaller* than distance PP$''$ at P''_r in P$''$ - equivalently, $\mathrm{PP}''(P''_r)$ was *greater* than $\mathrm{PP}'(P'_r)$ - iff both reflection events P'_r & P''_r were caused by the same event of emission P_e in P, but caused successive events of absorption P_a & $P_{a'}$ in P, so that a signal emitted at P_a, when reflected at P'''_r by P$'''$ at distance $\mathrm{PP}'''(P'''_r) \neq 0$, arrived at $P_{a'}$.

Df.8 *Permamently Smaller / Greater Distance.*

$\mathrm{PP}' < \mathrm{PP}'' \,.\equiv.\, \forall P'_r, P''_r : \mathrm{PP}'(P'_r) < \mathrm{PP}''(P''_r)$

Translation: PP$'$ remains smaller than PP$''$ iff it is smaller at all events of reflection.

Df.9 *Optical Lines.*

$\boldsymbol{opt}(P'_e \preceq P_a) \,.\equiv.\, P'_e \to \ldots \to P_a \wedge P'_e \to P_a$

Translation: The causal succession $P'_e \preceq P_a$ formed an optical line iff two signals beginning at the same emission event $P'_e \in \mathrm{P}'$: the indirect one $P'_e \preceq P_a$, and the direct one $P'_e \to P_a$, both terminated at the same absorbtion event $P_a \in \mathrm{P}$; i.e. the optical line was observed at $P_a \in \mathrm{P}$.

Df.10 *Light-Cones.*

$\boldsymbol{cone}(P_a) \,.\equiv.\, \{P^i_e | \boldsymbol{opt}(P^i_e \preceq P_a)\}$

Translation: The light-cone observed at $P_a \in \mathrm{P}$ was the set of all optical lines terminating at P_a.

Comment: The only way of determing the top angle of past cones is by arbitrary convention; this corresponds to the fact that, in ***SR***, "the one-way light-speed" depends on convention.

Df.11 *Observational Perspectives.*

$\boldsymbol{persp}(\mathrm{P}) \,.\equiv.\, \{P_a | \boldsymbol{cone}(P_a)\}$

Translation: The observational perspective of P was the set of all light cones terminating in P.
Comment: an observational perspective frames the ***world-view*** of an observer, cf. ch5 §7.

Df.12 *Spheres.*

$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \equiv \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}' \neq 0\}$

Translation: A sphere (a spherical surface) is the set (the geometric site) of all those points that keep the same distance to a given fix-point P as does another given point P', different from P.
Comment: If we allowed $\mathrm{PP}' = 0$, the sphere $\boldsymbol{S}(\mathrm{P},\mathrm{P}')$ would degenerate into a point.

Th.1 *Non-identity of Spheres.*

$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \neq \boldsymbol{S}(\mathrm{P}',\mathrm{P})$

Translation: A sphere about P with radius PP' differs from a sphere about P' with radius $\mathrm{P}'\mathrm{P}$.
Proof: $\forall \mathrm{P}, \forall \mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow .\, \boldsymbol{S}(\mathrm{P},\mathrm{P}') = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}'\} \neq \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{P}'\mathrm{P}\} = \boldsymbol{S}(\mathrm{P}',\mathrm{P})$

Df.13 *Similarity of Spheres.*

$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}'',\mathrm{P}''') \,.\equiv.\, \forall \mathrm{P}^i, \forall \mathrm{P}^j : \mathrm{PP}^i = \mathrm{PP}' = \mathrm{P}''\mathrm{P}''' = \mathrm{P}''\mathrm{P}^j$

Translation: Spheres are similar if their defining points are equidistant, i.e. their radii equal.

Cor.1 *Similarity of Spheres is Reciprocal.*

$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}',\mathrm{P})$

Translation: The similarity of different spheres with equal radii is reciprocal.
Proof: Follows immediately from the preceding definition of similarity.

Cor.2 *Similarity of Spheres is Transitive.*

$\boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}',\mathrm{P}'') \,\wedge\, \boldsymbol{S}(\mathrm{P}',\mathrm{P}'') \simeq \boldsymbol{S}(\mathrm{P}'',\mathrm{P}''') \,.\Rightarrow.\, \boldsymbol{S}(\mathrm{P},\mathrm{P}') \simeq \boldsymbol{S}(\mathrm{P}'',\mathrm{P}''')$

Translation: The similarity of different spheres with equal radii is transitive.
Proof: Follows immediately from the preceding definition of similarity.

Df.14 *Circles.*

$\boldsymbol{C}(\mathrm{P},\mathrm{P}') \equiv \{\boldsymbol{S}(\mathrm{P},\mathrm{P}') \cap \boldsymbol{S}(\mathrm{P}',\mathrm{P})\}$

Translation: A circle is definable as the intersection between two similar spheres.
Comment: So, if the radius of $\boldsymbol{C}(\mathrm{P},\mathrm{P}')$ were measurable, it would be $|\mathrm{PP}'|/\sqrt{2}$.

Th.2 *Circles as Sets.*

$\boldsymbol{C}(\mathrm{P},\mathrm{P}') = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\}$

Translation: A circle is the set of all points P^i keeping the same distance PP' to both P and P'.
Proof: $\forall \mathrm{P}, \forall \mathrm{P}', \forall \mathrm{P}'' : \mathrm{P} \neq \mathrm{P}' \neq \mathrm{P}^i \Rightarrow \{\boldsymbol{S}(\mathrm{P},\mathrm{P}') \cap \boldsymbol{S}(\mathrm{P}',\mathrm{P})\} = \big\{\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}'\} \cap$
$\{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{P}'\mathrm{P}\}\big\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}' \wedge \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\}$

Df.15 *Planes.*

$\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') \equiv \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i \neq 0\}$

Translation: A plane is the set of all points which are equidistant from two given points, $\mathrm{P} \neq \mathrm{P}'$.
Comment: The defining points do not belong to the plane defined, i.e.: $\mathrm{P},\mathrm{P}' \notin \boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$

Th.3 *Identity of Planes.*

$\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') = \boldsymbol{Pl}(\mathrm{P}',\mathrm{P})$

Translation: The plane between P and P′ is identical to the plane between P′ and P.
Proof: $\forall\,\mathrm{P},\forall\,\mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow .\,\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{PP}^i\}$

Th.4 *Circles as Subsets of Planes.*

$\boldsymbol{C}(\mathrm{P},\mathrm{P}') \subset \boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$

Translation: The circle defined by P,P' is a subset of the plane defined by P,P'.
Proof: $\{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}' \,.\Rightarrow.\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}'\} \subset$
$\{\mathrm{P}^i \,|\, \mathrm{P} \neq \mathrm{P}'. \Rightarrow .\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}' \vee \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i \neq \mathrm{PP}'\}$

' Th.5 *Circles as Intersections of Spheres and Planes.*

$\boldsymbol{C}(\mathrm{P},\mathrm{P}') \,.=.\, \boldsymbol{S}(\mathrm{P},\mathrm{P}') \cap \boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$

Translation: The sphere $\boldsymbol{S}(\mathrm{P},\mathrm{P}')$ and the plane $\boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$ intersect in the circle $\boldsymbol{C}(\mathrm{P},\mathrm{P}')$.
Proof: $\forall\,\mathrm{P},\forall\,\mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow .\,\big\{\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}'\} \cap \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\}\big\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{PP}' = \mathrm{P}'\mathrm{P}^i\}$

Df.16 *Straight Lines.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \equiv \{\mathrm{P}^i \,|\, \mathrm{PP}'' < \mathrm{PP}'\mathrm{P}'' \wedge \mathrm{P}'\mathrm{P}'' < \mathrm{P}'\mathrm{PP}'' \wedge \mathrm{P}'\mathrm{P} < \mathrm{P}'\mathrm{P}''\mathrm{P} \,.\Rightarrow.\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\}$

Translation: A line is the set of all points equidistant from three given points forming a triangle.
Comment: The defining points do not belong to the line defined, i.e.: $\mathrm{P},\mathrm{P}',\mathrm{P}'' \notin \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'')$. The definition yields a measure for the curvature (deviation from straightness) of optical lines.

Th.6 *Identity of Lines.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') = \boldsymbol{Lin}(\mathrm{P}',\mathrm{P}'',\mathrm{P}) = \boldsymbol{Lin}(\mathrm{P}'',\mathrm{P},\mathrm{P}') =$
$= \boldsymbol{Lin}(\mathrm{P}'',\mathrm{P}',\mathrm{P}) = \boldsymbol{Lin}(\mathrm{P}',\mathrm{P},\mathrm{P}'') = \boldsymbol{Lin}(\mathrm{P},\mathrm{P}'',\mathrm{P}')$

Translation: The line defined by $\mathrm{P},\mathrm{P}',\mathrm{P}''$ is identical to that defined by $\mathrm{P}',\mathrm{P}'',\mathrm{P}$ which is identical to the one defined by $\mathrm{P}'',\mathrm{P},\mathrm{P}'$, i.e., a line is indifferent to the order of its defining points.
Proof: $\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i = \mathrm{PP}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}''\mathrm{P}^i = \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} =$
$= \{\mathrm{P}^i \,|\, \mathrm{P}''\mathrm{P}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{PP}^i\} = \{\mathrm{P}^i \,|\, \mathrm{P}'\mathrm{P}^i = \mathrm{PP}^i = \mathrm{P}''\mathrm{P}^i\} = \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}''\mathrm{P}^i = \mathrm{P}'\mathrm{P}^i\}$

Th.7 *Intersecting Planes.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') = \{\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') \cap \boldsymbol{Pl}(\mathrm{P},\mathrm{P}'')\}$

Translation: The line defined by equidistance to $\mathrm{P},\mathrm{P}',\mathrm{P}''$ is identical to the intersection of the plane defined by equidistance to P,P' and the plane defined by equidistance to P,P''.
Proof: $\forall\,\mathrm{P},\mathrm{P}' : \mathrm{P} \neq \mathrm{P}' \Rightarrow .\,\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\} = \big\{\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} \cap \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}''\mathrm{P}^i\}\big\}$

Th.8 *Lines as Subsets of Planes.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \subset \boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$

Translation: The line defined by $\mathrm{P},\mathrm{P}',\mathrm{P}''$ is a subset of the plane defined by P,P'.

Proof: $\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\} \subset \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i \vee \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i \neq \mathrm{P}''\mathrm{P}^i\}$

Th.9 *Planes as Sets of Lines.*

$\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') = \{\mathrm{P}''|\, \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'')\}$

Translation: $\boldsymbol{Pl}(\mathrm{P},\mathrm{P}')$ is identical to the set of all lines $\{\mathrm{P}''|\, \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'')\}$.

Proof: $\{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i\} =$

$\{\mathrm{P}^i \,|\, \forall\, \mathrm{P}'' : \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\}\} = \{\mathrm{P}'' \,|\, \{\mathrm{P}^i \,|\, \mathrm{PP}^i = \mathrm{P}'\mathrm{P}^i = \mathrm{P}''\mathrm{P}^i\}\}$

Df.17 *Parallellity of Planes.*

$\boldsymbol{Pl}(\mathrm{P},\mathrm{P}') \parallel \boldsymbol{Pl}(\mathrm{P}'',\mathrm{P}''') \,.\equiv.\, \exists\, \boldsymbol{Lin}(\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3) : \mathrm{P},\mathrm{P}',\mathrm{P}'',\mathrm{P}''' \in \boldsymbol{Lin}(\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3)$

Translation: Two planes are said to be parallel iff their defining points belong to the same line.

Df.18 *Parallellity of Lines.*

$\boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \parallel \boldsymbol{Lin}(\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3) \,.\equiv.\, \exists\, \boldsymbol{Pl}(\mathrm{P}^a,\mathrm{P}^b) : \mathrm{P},\mathrm{P}',\mathrm{P}'',\mathrm{P}^1,\mathrm{P}^2,\mathrm{P}^3 \in \boldsymbol{Pl}(\mathrm{P}^a,\mathrm{P}^b)$

Translation: Two lines are said to be parallel iff their defining points belong to the same plane.

Df.19 *Perpendicularity.*

$\boldsymbol{Pl}(\mathrm{P}^1,\mathrm{P}^2) \perp \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \,.\equiv.\, \mathrm{P}^1,\mathrm{P}^2 \in \boldsymbol{Lin}(\mathrm{P},\mathrm{P}',\mathrm{P}'') \vee \mathrm{P},\mathrm{P}',\mathrm{P}'' \in \boldsymbol{Pl}(\mathrm{P}^1,\mathrm{P}^2)$

Translation: The plane defined by $\mathrm{P}^1,\mathrm{P}^2$ is said to be perpendicular to the line defined by $\mathrm{P},\mathrm{P}',\mathrm{P}''$ (not collinear) iff either: $a)$ the points $\mathrm{P}^1,\mathrm{P}^2$ belong to the line defined by $\mathrm{P},\mathrm{P}',\mathrm{P}''$, or: $b)$ the points $\mathrm{P},\mathrm{P}',\mathrm{P}''$ belong to the plane defined by $\mathrm{P}^1,\mathrm{P}^2$, or: *c*) both $a)$ and $b)$.

Df.20 *The Substrate.*

$\boldsymbol{Subst} \equiv \{\, \mathrm{P}^i \,|\, \forall\, \mathrm{P},\mathrm{P}',\mathrm{P}'',P_r',P_r'' : \; \mathrm{PP}'(P_r') = \mathrm{PP}''(P_r'') \Rightarrow \mathrm{PP}' = \mathrm{PP}''\}$

Translation: The Substrate is a set of points/particles characterized by the following property: if two points in the Substrate ever keep the same distance to a third one, they always do so.

Df.21 *Fundamental Particles* (FP).

$\mathrm{P} = \mathrm{P}_F \,.\equiv.\, \mathrm{P} \in \boldsymbol{Subst}$

Translation: The particle P is called fundamental, P_F, iff it belongs to the Substrate.

Comment: All *FP* are said to be *at rest* in the Substrate.

Df.22 *Accidental Particles* (AP).

$\mathrm{Q} = \mathrm{Q}_{\boldsymbol{A}} \,.\equiv.\, \mathrm{Q} \notin \boldsymbol{Subst}$

Translation: The particle Q is called accidental, $\mathrm{Q}_{\boldsymbol{A}}$, iff it does not belong to the Substrate.

Comment: All *AP* are said to be *in motion* in the Substrate.

AX.3 *The Substrate is not Empty.*

$\exists\, \mathrm{P},\mathrm{P}',\mathrm{P}'',\mathrm{P}''' : \mathrm{P},\mathrm{P}',\mathrm{P}'',\mathrm{P}''' \in \boldsymbol{Subst} \,.\wedge.\, \mathrm{PP}' \neq \mathrm{P}'\mathrm{P}'' \neq \mathrm{P}''\mathrm{P}'''$

$.\wedge.\; \forall\, \boldsymbol{Pl}(\mathrm{P}_1,\mathrm{P}_2) : \mathrm{P},\mathrm{P}',\mathrm{P}'' \in \boldsymbol{Pl}(\mathrm{P}_1,\mathrm{P}_2) \;\Rightarrow\; \mathrm{P}'''' \notin \boldsymbol{Pl}(\mathrm{P}_1,\mathrm{P}_2)$

Translation: The Substrate contains four non-equidistant *FP* not belonging to the same plane.

Df.23 *Midway Particles.*
$\mathrm{P} = \boldsymbol{m}(\mathrm{P}^1, \mathrm{P}^2) \,.\equiv.\, \mathrm{PP}^1 = \mathrm{PP}^2 \wedge \mathrm{P}^1\mathrm{P}^2 = \mathrm{P}^1\mathrm{PP}^2$
Translation: P is the midway particle of $\mathrm{P}^1$and P^2 iff $\mathrm{P}^1\mathrm{P}^2$ equals $\mathrm{P}^1\mathrm{PP}^2$ and PP^1 equals PP^2.

AX.4 *The Substrate is Dense.*
$\forall\, \mathrm{P}^1, \mathrm{P}^2 : \mathrm{P}^1, \mathrm{P}^2 \subset \boldsymbol{Subst} \,.\Rightarrow \exists\, \mathrm{P} : \mathrm{P} = \boldsymbol{m}(\mathrm{P}^1, \mathrm{P}^2) \in \boldsymbol{Subst}$
Translation: Any two fundamental particles have a midway particle which is also fundamental.

AX.5 *The Substrate is Infinite.*
$\forall\, \mathrm{P}^1, \mathrm{P}^2 : \mathrm{P}^1, \mathrm{P}^2 \subset \boldsymbol{Subst} \,.\Rightarrow \exists\, \mathrm{P}^3 : \mathrm{P}^3 \in \boldsymbol{Subst} \wedge \mathrm{P}^2 = \boldsymbol{m}(\mathrm{P}^1, \mathrm{P}^3)$
Translation: For any two FPs there is a 3rd FP so that the 2nd is midway between 1st & 3rd.

AX.6 *Coincidence entails Collapse.*
$\forall\, \mathrm{P}, \mathrm{P}', \mathrm{P}'', P_r'' : \mathrm{P}, \mathrm{P}', \mathrm{P}'' \in \boldsymbol{Subst} \,\Rightarrow .\, \mathrm{PP}''(P_r'') = 0 \Rightarrow \mathrm{P}'\mathrm{P}''(P_r'') = 0$
Translation: It holds for any three members of the Substrate that if two of them ever coincide at an instant they all coincide at that instant so that the entire Substrate collapses into a singularity.
Comment: An expanding and accelerating universe may not stem from an initial singularity.

Df.24 *Isotropy of Perspectives.*
$\boldsymbol{iso\text{-}persp}(\mathrm{P}) \,.\equiv.\, \boldsymbol{persp}(\mathrm{P}) \,.\wedge.\, \forall\, \mathrm{P}' : \boldsymbol{persp}(\mathrm{P}) \cap \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \,.=.\, \boldsymbol{C}(\mathrm{P}, \mathrm{P}')\}$
Translation: The world-view $\boldsymbol{persp}(\mathrm{P})$ of an observer P is *isotropic* iff, for all other particles P', the intersection of $\boldsymbol{persp}(\mathrm{P})$ and $\boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$ is the circle $\boldsymbol{C}(\mathrm{P}, \mathrm{P}')$ with center $\boldsymbol{m}(\mathrm{P}, \mathrm{P}')$.
Comment: If the $\boldsymbol{iso\text{-}persp}(\mathrm{P})$ of P is cut by a plane defined by P and some other particle P' indicating an arbitrary direction in space, the intersection is always circular, not elliptical.

Cor.3 *Isotropic Perspectives are Sets of Concentric Spheres.*
$\boldsymbol{iso\text{-}persp}(\mathrm{P}) \,.=.\, \{\mathrm{P}^i \,|\, \mathrm{P}^i \in .\, \boldsymbol{persp}(\mathrm{P}) \cap \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \,.\Rightarrow.\, \mathrm{P}^i \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}')\}$
Translation: The isotropic perspective of an observer P is the set of all spheres centered in P.
Proof: By df.24, $\boldsymbol{persp}(\mathrm{P}) \cap \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \,.=.\, \boldsymbol{C}(\mathrm{P}, \mathrm{P}')$, and by df.14, $\boldsymbol{C}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{S}(\mathrm{P}, \mathrm{P}')$

AX.7 *The Substrate is Isotropic.*
$\forall\, \mathrm{P} :\ \mathrm{P} \in \boldsymbol{Subst} \,\Rightarrow .\, \boldsymbol{persp}(\mathrm{P}) = \boldsymbol{iso\text{-}persp}(\mathrm{P})$
Translation: The observational perspective of any fundamental particle P is isotropic.
Comment: The Substrate fulfils the *Principle of Cosmic Isotropy*, cf. Cusanus [ca.1450].

Th.10 *Isotropy of Light-Cones.*
$\forall\ \mathrm{P}, \mathrm{P}', \mathrm{P}^{\iota} : \mathrm{P}, \mathrm{P}', \mathrm{P}^{\iota} \in \boldsymbol{Subst} : \,:\, \Rightarrow \,:\, : \forall\, P_e, P_r^{\iota}, P_a : P_e \preceq P_r^{\iota} \preceq P_a \,.\wedge.$
$P_r^{\iota} \in \boldsymbol{cone}(P_a) \subset \boldsymbol{persp}(\mathrm{P}) :\, \Rightarrow\, : \mathrm{P}^{\iota} \subset \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \,.\Rightarrow.\, \mathrm{P}^{\iota} \subset \boldsymbol{C}(\mathrm{P}, \mathrm{P}')\}$
Translation: For all fundamental particles P, P', and $\mathrm{P}^{\iota} = \{\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, ..\}$ it holds that if signals emitted at $P_e \in \mathrm{P}$ and absorbed $P_a \in \mathrm{P}$ are reflected at events P_e^{ι} contained in different particles P^{ι}, equidistant to P, then if the particles P^{ι} belong to $\boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$ they also belong to $\boldsymbol{C}(\mathrm{P}, \mathrm{P}')$.
Proof: By Ax.7, th.9, and instantiation of df.24.

AX.8 *Space has Three Dimensions.*

$\forall \mathrm{P}', \mathrm{P}'', \mathrm{P}''' : \mathrm{P}', \mathrm{P}'', \mathrm{P}''' \in \boldsymbol{Subst} \wedge \mathrm{P}'\mathrm{P}'' = \mathrm{P}''\mathrm{P}''' = \mathrm{P}'''\mathrm{P}'$

$\Rightarrow \; : \exists 2\,\mathrm{P} : \mathrm{P} \in \boldsymbol{Subst} \wedge \mathrm{PP}' = \mathrm{PP}'' = \mathrm{PP}''' = \mathrm{P}'\mathrm{P}''$

Translation: For any three equidistant particles, members of the Substrate, there are two and only two more particles, members of the Substrate, that remain equidistant from the other three.
Comment: The difference between the two tetrahedra is the *origin af handedness* in 3-space.

AX.9 *Signals between AP follow tracks between FP.*

$\forall Q_e', Q_a'' : Q_e' \preceq Q_a'' \; : \; \Rightarrow \; : \exists \mathrm{P}', \mathrm{P}'' : \mathrm{P}', \mathrm{P}'' \in \boldsymbol{Subst} \wedge \mathrm{P}'\mathrm{Q}'(Q_e') = \mathrm{P}''\mathrm{Q}''(Q_a'') = 0$

Translation: If the event Q_e' in Q′ caused the event Q_a'' in Q″, then there were FPs P′ and P″, so that Q′ coincided with P′ at Q_e', and Q″ coincided with P″ at Q_a''
Comment: The Substrate serves as an *aether*, i.e. a "medium" for the transmission of "photons"; thus "light-tracks" (if describable at all) can be described by reference to fundamental particles.

Df.25 *Flatness of Space.*

$\forall \mathrm{P}^0, \forall \mathrm{P}^1, \forall \mathrm{P}^2, \forall \mathrm{P}^3, \forall \mathrm{P}^4, \forall \mathrm{P}^5, \forall \mathrm{P}^6 : \mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6 \in \boldsymbol{Subst} \; : \; \Rightarrow \; :$

$\mathrm{P}^0\mathrm{P}^1 = \mathrm{P}^0\mathrm{P}^2 = \mathrm{P}^0\mathrm{P}^3 = \mathrm{P}^0\mathrm{P}^4 = \mathrm{P}^0\mathrm{P}^5 = \mathrm{P}^0\mathrm{P}^6 = \mathrm{P}^1\mathrm{P}^2 = \mathrm{P}^2\mathrm{P}^3 = \mathrm{P}^3\mathrm{P}^4 = \mathrm{P}^4\mathrm{P}^5 = \mathrm{P}^5\mathrm{P}^6 = \mathrm{P}^6\mathrm{P}^1$

$\Rightarrow \; . \; \exists \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') : \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst} \; \wedge \; \mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6 \in \boldsymbol{Pl}(\mathrm{P}, \mathrm{P}')$

Translation: Space is flat, or Euclidean, iff for all particles $\mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6$, members of the Substrate, if $\mathrm{P}^1, \mathrm{P}^2, \mathrm{P}^3, \mathrm{P}^4, \mathrm{P}^5, \mathrm{P}^6$ form a regular hexagon with P^0 in its center, all distances between neighbouring corners being equal to their distances (radii) from the center, then there is a plane, subset of the Substrate, to which they all belong, the six corners as well as their center.
Comment: This definition leaves the issue open whether space is Euclidean or non-Euclidean. If the flat hexagon does not fit into a plane, leaving open areas, the plane (space) is *hyperbolic*. If the hexagon does not fit into a plane, due to overlapping areas, the plane (space) is *spherical*. This is explainable even to children by comparing flat to saddle-like and ball-shaped surfaces.

Df.26 *Relative Simultaneity of Events* (P-*simultaneity*).

$\boldsymbol{sim}_{\mathrm{P}}(P_r', P_r'') \,.\equiv.\, \mathrm{P}, \mathrm{P}', \mathrm{P}'' \in \boldsymbol{Subst} \,.\wedge.\, \mathrm{PP}'(P_r') = \mathrm{PP}''(P_r'')$

Translation: Two events, P_r' in P′ and P_r'' in P″, are P-simultaneous with respect to observer P, iff P, P′, P″ all belong to the Substrate and P′ at P_r' is equidistant with P″ at P_r'', relative to P.

Th.11 P-*simultaneity is Reciprocal between the Points on any Sphere centered at* P.

$\forall P_r', \forall P_r'' : \mathrm{P}'' \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst} \,.\Rightarrow\; : \boldsymbol{sim}_{\mathrm{P}}(P_r', P_r'') \Leftrightarrow \boldsymbol{sim}_{\mathrm{P}}(P_r'', P_r')$

Translation: The event P_r' is P-simultaneous with the event P_r'' iff P_r'' is P-simultaneous with P_r'.
Proof: $\mathrm{P}'' \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \,.\Rightarrow\; : \mathrm{PP}' = \mathrm{PP}'' \Rightarrow . \,\mathrm{PP}'(P_r') = \mathrm{PP}''(P_r'')$

Th.12 P-*Simultaneity is Transitive between the Points on any Sphere centered at* P.

$\forall P_r', \forall P_r'', \forall P_r''' : \mathrm{P}'', \mathrm{P}''' \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst} \,.\Rightarrow\; :$

$\boldsymbol{sim}_{\mathrm{P}}(P_r', P_r'') \Leftrightarrow \boldsymbol{sim}_{\mathrm{P}}(P_r'', P_r''') \Leftrightarrow \boldsymbol{sim}_{\mathrm{P}}(P_r''', P_r')$

Translation: If the FPs P″, P‴ belong to the sphere centered at P and defined by FPs P, P′, then: P_r' is P-sim. with P_r'' iff P_r'' is P-sim. with P_r''' iff P_r''' is P-sim. with P_r'.
Proof: $\mathrm{P}'', \mathrm{P}''' \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \,.\Rightarrow\; : \mathrm{PP}' = \mathrm{PP}'' = \mathrm{PP}''' \Rightarrow . \,\mathrm{PP}'(P_r') = \mathrm{PP}''(P_r'') = \mathrm{PP}'''(P_r''')$

Comment: By increasing distances PP′, PP″, PP‴ *ad inf.*, the transitivity of P-simultaneity can be generalized to cover all FP in the substrate being situated on approximately flat surfaces.

Th.13 P-*Simultaneity is Transitive between Equidistant Points on a Straight Line.*

$$\forall \mathrm{P}^0, \forall \mathrm{P}^1, \forall \mathrm{P}^{-1}, ..., \forall \mathrm{P}^{n-1}, \forall \mathrm{P}^{1-n}, \forall \mathrm{P}^{n}, \forall \mathrm{P}^{-n}, ... : \;:$$

$$\exists\, \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') : \mathrm{P}^0, \mathrm{P}^1, \mathrm{P}^{-1}, ..., \mathrm{P}^{n-1}, \mathrm{P}^{1-n}, \mathrm{P}^{n}, \mathrm{P}^{-n}, ... \in \boldsymbol{Lin}(\mathrm{P}', \mathrm{P}'', \mathrm{P}''') \subset \boldsymbol{Subst}$$

$$: \;\Rightarrow\; : ... = \mathrm{P}^{-n}\mathrm{P}^{1-n} = ... = \mathrm{P}^{-1}\mathrm{P}^{0} = \mathrm{P}^{0}\mathrm{P}^{1} = ... = \mathrm{P}^{n-1}\mathrm{P}^{n} = ... \,.\Rightarrow .$$

$$\boldsymbol{sim}_{\mathrm{P}^0}(P_1^{-1}, P_1^{1}) \,.\underset{2}{\wedge}.\, \boldsymbol{sim}_{\mathrm{P}^0}(P_2^{-2}, P_2^{2}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_2^{-2}, P_2^{0}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_2^{0}, P_2^{2})$$

$$.\underset{3}{\wedge}.\, \boldsymbol{sim}_{\mathrm{P}^0}(P_3^{-3}, P_3^{3}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_3^{-1}, P_3^{1}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_3^{-3}, P_3^{-1}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_3^{1}, P_3^{3}) \,.\underset{4}{\wedge}.$$

$$\boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-4}, P_4^{4}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-4}, P_4^{-2}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-2}, P_4^{0}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{0}, P_4^{2}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{2}, P_4^{4}) \wedge \boldsymbol{sim}_{\mathrm{P}^0}(P_4^{-2}, P_4^{2}) \,.\underset{5}{\wedge}\, ...$$

Translation: Simultaneity is transitive between all equidistant particles on a line relative to an arbitrarily chosen midway particle P^0, counting right units as positive and left units as negative.

Proof: $\mathrm{P}^0\mathrm{P}^{-1}(P_1^{-1}) = \mathrm{P}^0\mathrm{P}^1(P_1^1) \underset{2}{\wedge} \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}(P_2^{-2}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0(P_2^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^0(P_2^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2(P_2^2) \underset{5}{\wedge}$
$\mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-3}(P_3^{-3}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-1}(P_3^{-1}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0\mathrm{P}^{-1}(P_3^{-1}) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^0\mathrm{P}^1(P_3^1) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^1(P_3^1) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3(P_3^3) \underset{4}{\wedge}$
$\mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-3}\mathrm{P}^{-4}(P_4^{-4}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^{-2}\mathrm{P}^{-3}\mathrm{P}^{-2}(P_4^{-2}) = \mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0\mathrm{P}^{-1}\mathrm{P}^0(P_4^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^0\mathrm{P}^1\mathrm{P}^0(P_4^0) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3\mathrm{P}^2(P_4^2) = \mathrm{P}^0\mathrm{P}^1\mathrm{P}^2\mathrm{P}^3\mathrm{P}^4(P_4^4)$

Comment: By inserting more and more midway points we can refine the partitioning to pursue the construction of a network which can cover any point on the line with any wanted precision. This also holds for those radii which are the centerlines of the isotropic perspectives of FPs.

Scholium. Combining theorems 10-12, P-simultaneity is definable for the entire Substrate. However only a finite number of the points contained in a line can be covered by our procedure. In order to ensure simultaneity for all points on a line we are therefore faced with the choice between postulating atoms of time, thus also of space, or postulating simultaneity to be absolute. It turns out, however, that assuming simultaneity to be absolute does *not* prevent moving clocks from appearing retarded, as evidenced by experiment in agreement with standard relativity.

AX.10 *Simultaneity is Absolute in the Substrate.*

$$\forall \mathrm{P}', \mathrm{P}'', \mathrm{P}''', \exists\, \mathrm{P} : \mathrm{P}'', \mathrm{P}''' \in \boldsymbol{S}(\mathrm{P}, \mathrm{P}') \subset \boldsymbol{Subst} \,.\wedge.$$

$$\forall P_r', \exists 1 P_r'', \exists 1 P_r''' : \boldsymbol{sim}_{\mathrm{P}}(P_r', P_r'') \Leftrightarrow \boldsymbol{sim}_{\mathrm{P}}(P_r'', P_r''') \Leftrightarrow \boldsymbol{sim}_{\mathrm{P}}(P_r''', P_r')$$

Translation: For any triplet of particles in the Substrate, and for any event on a member of this triplet, there is one and only one event on each of the two other members of the triplet which is simultaneous with the first event, as judged from a permanent center of the triplet.

Comment: From ax.s 9 & 10 taken together it follows that P-simultaneity is transitive between events in the entire substrate, i.e., between events contained in AP as well as in FP.

Df.27 *Zig-zag Signals.*

$$\boldsymbol{zzs}(\mathrm{P}, \mathrm{P}') \,.\equiv.\, P_0 \to P_0' \to P_1 \to P_1' \to P_2 \to P_2' \,...$$

Translation: There was a zig-zag signal between P and P′ iff P_i in P and P_i' in P′ formed a chain of events $P_0, P_0', P_1, P_1', P_2, P_2'$ following each other in immediate causal succession.

Df.28 *Cosmic Clocks.*

$$\boldsymbol{ccl}(\mathrm{PP}') \,.\equiv.\, \exists\, \boldsymbol{zzs}(\mathrm{P}, \mathrm{P}') : \mathrm{P}, \mathrm{P}' \in \boldsymbol{Subst} \wedge$$

$$\{..., P_0, P_1, P_2, ...\} \rightharpoonup \{..., T(P_0), T(P_1), T(P_2), ...\} \subseteq \boldsymbol{\mathcal{R}}$$

Translation: The cosmic clock carried by P **is** a Langevin clock with unit PP′P defined by a zig-zag signal exchanged between P & P′, P & P′ being both FPs, and successive events P_i being mapped on instants $T(P_i)$ constituting a subset of the set of real numbers $\boldsymbol{\mathcal{R}}$.

Th.14 *There are Cosmic Clocks.*

$\forall\,\mathrm{P}, \forall\,\mathrm{P}' : \mathrm{P}, \mathrm{P}' \in \boldsymbol{Subst} \,.\Rightarrow\; : \exists\, \boldsymbol{ccl}(\mathrm{PP}')$

Translation: For any pair P, P′ of FPs we can construct the cosmic clock $\boldsymbol{ccl}(\mathrm{PP}')$.
Proof: Follows immediately from ax.s 1&10 together with df.28.

Df.29 *Non-Identity of Cosmic Clocks.*

$\boldsymbol{ccl}(\mathrm{PP}') \neq \boldsymbol{ccl}(\mathrm{P}'\mathrm{P})$

Translation: Two cosmic clocks are different iff their carriers are different.

Df.30 *Similarity of Cosmic Clocks.*

$\boldsymbol{ccl}(\mathrm{PP}') \simeq \boldsymbol{ccl}(\mathrm{P}''\mathrm{P}''') \,.\equiv.\, \mathrm{PP}' = \mathrm{P}''\mathrm{P}'''$

Translation: Two cosmic clocks are similar iff their unit distances are equal.

Cor.4 *Similarity of Cosmic Clocks is Reciprocal.*

$\forall\,\mathrm{P}, \forall\,\mathrm{P}' : \boldsymbol{ccl}(\mathrm{PP}') \simeq \boldsymbol{ccl}(\mathrm{P}'\mathrm{P})$

Translation: Two different cosmic clocks defined by the same pair of particles are similar.
Proof: Follows immediately from their defining unit distances being equal.

Cor.5 *Similarity of Cosmic Clocks is Transitive.*

$\forall\,\mathrm{P}, \forall\,\mathrm{P}', \forall\,\mathrm{P}'', \forall\,\mathrm{P}''' :$

$\boldsymbol{ccl}(\mathrm{PP}') \simeq \boldsymbol{ccl}(\mathrm{P}'\mathrm{P}'') \,\wedge\, \boldsymbol{ccl}(\mathrm{P}'\mathrm{P}'') \simeq \boldsymbol{ccl}(\mathrm{P}''\mathrm{P}''') \,.\Rightarrow.\, \boldsymbol{ccl}(\mathrm{PP}') \simeq \boldsymbol{ccl}(\mathrm{P}''\mathrm{P}''')$

Translation: Similarity is transitive between any triple of cosmic clocks.
Proof: Follows immediately from their defining unit distances being equal.

Df.31 *Signal-Functions.*

$T^{\mathrm{P}'} = \Theta^{\mathrm{PP}'}(T^{\mathrm{P}}). \equiv .\, T(P_i') = \Theta^{\mathrm{PP}'}(T(P_{i-1})). \equiv .\, \exists\, \boldsymbol{zzs}(\mathrm{P}, \mathrm{P}') : T(P_i) \rightharpoonup T(P_j')$

Translation: There is a signal-function from instants $T(P_{i-1})$ on the clock $\boldsymbol{ccl}(\mathrm{PP}')$ carried by P to instants $T(P_{\mathrm{i}}')$ on the clock $\boldsymbol{ccl}(\mathrm{P}'\mathrm{P})$ carried by P′ iff there is a zig-zag signal between P & P′ together with a mapping of preceding instants $T(P_{i-1})$ in P to succeding instants $T(P_{\mathrm{i}}')$ in P′.

Df.32 *Mapping a Clock onto Itself.*

$\boldsymbol{ccl}(\mathrm{PP}') \rightharpoonup \boldsymbol{ccl}(\mathrm{PP}') \,.\equiv.\, T_i^{\mathrm{P}} = \Theta^{\mathrm{P}'\mathrm{P}}\Theta^{\mathrm{PP}'}(T_{i-1}^{\mathrm{P}}). \equiv .\, T(P_i) = \Theta^{\mathrm{P}'\mathrm{P}}\big(\Theta^{\mathrm{PP}'}(T(P_{i-1}))\big)$

Translation: There is a mapping of the clock $\boldsymbol{ccl}(\mathrm{PP}')$ onto itself iff there is a zig-zag signal from P to P′ and back together with a signal-function from P to P′ mapping instants in P onto instants in P′and another signal-function from P′ to P mapping instants in P′ onto instants in P.

Df.33 *Congruence of Cosmic Clocks.*

$\boldsymbol{ccl}(\mathrm{PP}') \equiv \boldsymbol{ccl}(\mathrm{P}'\mathrm{P}) \,.\equiv.\, \Theta^{\mathrm{P}'\mathrm{P}} = \Theta^{\mathrm{PP}'} = \Theta$

Translation: The clock $\boldsymbol{ccl}(\mathrm{PP'})$ of P is said to be congruent with the clock $\boldsymbol{ccl}(\mathrm{P'P})$ of P′ iff the signal-function from P to P′ is identical to the signal-function from P′ to P.
Comment: Clocks are made congruent by *regraduation*, i.e. a formal adjustment of units & zero. A method for deriving the functional square root of $\Theta^{\mathrm{P'P}}\Theta^{\mathrm{PP'}}$ was found by Milne & Whitrow. That congruence of cosmic clocks is reciprocal follows at once from the definition just given. However, it was also shown by Whitrow & Milne that congruence of clocks is transitive among collinear particles iff their signal-functions commute.

Th.15 *Congruent Cosmic Clocks are Similar.*
$\forall\,\mathrm{P}, \forall\,\mathrm{P'} : \boldsymbol{ccl}(\mathrm{PP'}) \equiv \boldsymbol{ccl}(\mathrm{P'P})\,.\Rightarrow.\,\boldsymbol{ccl}(\mathrm{PP'}) \simeq \boldsymbol{ccl}(\mathrm{P'P})$
Translation: If two cosmic clocks are congruent they are also similar, i.e., keep the same rate.
Comment: Similar clocks keep the same rate. Congruent clocks also agree on a common zero.

Df.34 *Atomic Clocks.*
$\boldsymbol{acl}(\mathrm{Q}, r_a)\,.\equiv.\ \mathrm{Q} \in \boldsymbol{Universe} \wedge unit = r_a \wedge$
$\{..., Q_0, Q_1, Q_2, ...\} \rightharpoonup \{..., t(Q_0), t(Q_1), t(Q_2), ...\} \subset \boldsymbol{\mathcal{Z}}_{\pm}$
Translation: The atomic clock carried by Q is a mechanism devised to amplify the oscillations of atoms of a specified type displaying the natural frequency $1/r_a$, with events Q_i being mapped on instants $t(Q_i)$ constituting a very fine-grained subset of the set of natural numbers $\boldsymbol{\mathcal{Z}}_{\pm}$.

AX.11 *There are Atomic Clocks.*
$\forall\,\mathrm{Q} : \mathrm{Q} \in \boldsymbol{Universe}\,.\Rightarrow.\,\exists\,\boldsymbol{acl}(\mathrm{Q}, r_a)$
Translation: Any observer or particle in the universe is the carrier of at least one atomic clock.
Comment: This axiom differs somewhat from the other ten by its physical character.

Df.35 *Atomic Master Clocks.*
$\boldsymbol{mcl}(\mathrm{P}, r_a)\,.\equiv.\,\boldsymbol{acl}(\mathrm{P}, r_a) \wedge \mathrm{P} = \mathrm{P}_F \in \boldsymbol{Subst}$
Translation: A master clock is an atomic clock carried by a fundamental particle.

Df.36 *Atomic Slave Clocks.*
$\boldsymbol{scl}(\mathrm{Q}, r_a)\,.\equiv.\,\boldsymbol{acl}(\mathrm{Q}, r_a) \wedge \mathrm{Q} = \mathrm{Q}_A \notin \boldsymbol{Subst}$
Translation: A slave clock is an atomic clock carried by an accidental particle.

Df.37 *Similarity of Atomic Master Clocks.*
$\boldsymbol{mcl}(\mathrm{P}, r_a) \simeq \boldsymbol{mcl}(\mathrm{P'}, r'_a)\,.\equiv.\,r_a = r'_a = r_o$
Translation: Two atomic master clocks are similar (i.e. keep the same rate) iff they are both controlled by atoms of the same type displaying the same frequencies and having the same radii.
Comment: The slave clocks of accidental particles are never similar in the way defined here.

Df.38 *A Stationary Universe*
$\boldsymbol{Stat\text{-}Universe}\,.\equiv.\,\forall\,\boldsymbol{ccl}(\mathrm{PP'}), \exists\,\boldsymbol{mcl}(\mathrm{P}, r_a) : \Delta t/\Delta T \equiv const.$
Translation: The universe is stationary iff for any cosmic clock keeping the rate $1/\mathrm{PP'} = 1/\Delta T$ there is an atomic master clock with the natural rate $1/r_a = 1/\Delta t$, so that $\Delta t/\Delta T = const.$

Comment: This is equivalent to saying that the radii r_a of certain atoms are constant compared to some arbitrarily chosen cosmic distance PP′. Thus there is only one natural scale of time.

Df.39 *An Expanding Universe*

$\boldsymbol{Exp\text{-}Universe} \,.\equiv.\, \forall\, \boldsymbol{ccl}(\mathrm{PP}'), \exists\, \boldsymbol{mcl}(\mathrm{P}, r_a) : \mathit{\Delta} t/\mathit{\Delta} T \equiv \mathcal{R}(t) \equiv \mathcal{C}^{-1}(T)$

Translation: The universe is expanding iff for any cosmic clock with the rate $1/\mathrm{PP}' = 1/\mathit{\Delta} T$ there is an atomic master clock of rate $1/r_a = 1/\mathit{\Delta} t$, so that $\mathit{\Delta} t/\mathit{\Delta} T = \mathcal{R}(t) = \mathcal{C}^{-1}(T) \rightarrow \infty$.
Comment: This is equivalent to saying that the radii of atoms are steadily shrinking as compared to some arbitrarily chosen cosmic distance. So there are two natural time scales.

Df.40 *Frames Associated with Fundamental Particles .*

$\boldsymbol{Fra}(\mathrm{P}) \,.\equiv.\, \exists\, \{\mathrm{Q}_i \,|\, \mathrm{P} = \mathrm{P}_F \wedge \mathrm{PQ}_i = k_i\, r_a \wedge \boldsymbol{mcl}(\mathrm{P}, r_a) \underline{\equiv} \boldsymbol{scl}(\mathrm{Q}, r_a)\}$

Translation: The fundamental particle P is associated with a stationary reference frame iff P is surrounded by accidental particles Q_i at fixed distances to P as origo, counted by a finite number k_i of standard atomic radii r_a, each of these particles being provided with atomic slave clocks kept congruent to the original master clock of P which is situated in origo of P's frame.
Comment: If the universe is stationary, all such reference frames coincide with the Substrate. If, by contrast, the universe is expanding, it is the main point of our axiomatization that there are no such natural frames, and that their artificial construction would necessitate that the involved accidental particles were constrained by external forces to remain in their positions at fixed distances from the origo, and their associated slave clocks were constrained by external forces to remain congruent to the master clock in origo. *The substrate is the only true master frame.*

Df.41 *Frames Associated with Accidental Particles.*

$\boldsymbol{fra}(\mathrm{Q}) \,.\equiv.\, \exists\, \{\mathrm{R}_i \,|\, \mathrm{Q} = \mathrm{Q}_A \wedge \mathrm{QR}_i = k_i\, r_a \wedge \boldsymbol{scl}(\mathrm{Q}, r_a) \underline{\equiv} \boldsymbol{scl}(\mathrm{R}, r_a)\}$

Translation: The accidental particle Q is associated with a comoving reference frame iff Q is surrounded by accidental particles R_i keeping fixed distances to Q as origo, counted by a finite number k_i of standard atomic radii r_a, each of these particles being provided with atomic slave clocks kept congruent to the presumed master clock of Q and situated in origo of Q's frame.
Comment: If the universe is stationary, all such reference frames coincide with the Substrate. If, by contrast, the universe is expanding, it is our view that there are no such frames in nature, and that they can only be devised by imposing artificial constraints.

Cor.6 *Our Tempo-Spatial Geometry presupposes a Substratum.*

It is an immediate consequence of our entire approach, that our geometrical entities as well as their formal properties are only definable in a strict sense with explicit reference to the ideal of a universal substratum of fundamental particles and a superposed layer of accidental particles.

Scholium. An axiomatics giving priority to *tempo-spatial equidistance*, as outlined above, would seem to be of particular relevance to the biological ideas of Rowlands & Hill [2012].

=//=

3. CONCLUSION

Although this formal presentation is exceedingly sketchy, I think it allows us to affirm the conclusion of Walker [1959], that it is possible to assign clocks to all particles in the Substrate "which are not merely *congruent*, but also *equivalent* to each other", so that we henceforth have "the product structure $\mathcal{T} \times \mathcal{C}$ on the set of all events" - i.e. a *time-space* in the sense of Mercier - $\mathcal{T}$ being a temporal parameter - *a cosmic time* - and $\mathcal{C}$ being the space of particles.

However, Walker considered only one type of clocks, viz. those we have called cosmic. Further, Walker considered a single Substrate only, the Substrate being comparable to a unique "spray" in the sense of Schutz [1973]. We shall here follow Milne and Walker by assuming the Substrate to be singular and, indeed, unique. However, by introducing the atomic master clocks of fundamental particles we have, in contrast to Walker, also considered another type of clocks. This move is essential, since a comparison of cosmic and atomic clocks is necessary for making sense of an answer to the question whether the universe is stationary or expanding.

According to Eddington, "the theory of the expanding cosmos is equivalent to the theory of the shrinking atom". So it is impossible to decide whether the universe is expanding relative to the fixed sizes of its contents or whether its contents are shrinking relative to the dimensions of their stationary surroundings. It is obvious that different types of clocks, cosmic and atomic, are bound to measure spatial distances in different ways. This shows that Milne was right when he insisted that *the distinction between two basic scales of time, T & t, is mandatory* to physics. Now the principle of inertia, if valid at all, can only hold good relative to one of these scales. We shall follow Milne by assuming that it holds relative to the cosmic T-scale. Consequently we may expect that the free motion of particles is exposed to spontaneous accelerations when described relative to the atomic t-scale. This might be the key to explaining gravitation.

In fact, the phenomenon of gravitation can only emerge within an expanding universe. In such a universe the principle of Mach would finally be vindicated. However, it would hold the other way round than presumed by Mach and Einstein, as well as by their host of adherents: *by this turn gravity would be explained by inertia, instead of inertia being explained by gravity.* The Substrate, as described according to the atomic t-scale - to most people the only natural one since they don't perceive the atoms of their bodies to be shrinking - is a natural reference frame. Within this frame all fundamental particles and their associated master clocks are equivalent.

The position of an accidental particle Q is definable, relative to an arbitrary observer O, by the position of that particle $\mathrm{P} = \mathrm{P}_{FP}$ with which it momentarily coincides, just as its velocity is definable by the velocity of that particle $\mathrm{P}' = \mathrm{P}'_{FP}$ relative to which it is momentarily at rest. Now Q, on account of its motion relative to P, must possess a certain amount of kinetic energy. Shifting our point of view from choosing $\mathrm{O} = \mathrm{P}$ to choosing $\mathrm{O} = \mathrm{P}'$ this energy cannot vanish, all fundamental observers being equivalent; only it is no longer kinetic, but dynamic (potential). So, to Q, it is as if the gravitational potential of the whole universe were centered in P'!

In a kinematic universe no gravitational attraction holds between fundamental particles. Each single fundamental particle being a center of cosmic isotropy, all particles are equivalent; for this reason, fundamental particles are not themselves exposed to gravitational potentials; consequently. the identical master clocks of fundamental particles count the same cosmic time. Therefore we reject the cosmic differential field equations of Einstein and Friedmann.

LITERATURE

1. Cusanus, Nicolaus. [ca.1450]: *De Docta Ignorantia II, xii*, Herder Verlag:
Unde erit machina mundi quasi habens undique centrum et nullibi circumferentia, quoniam eius circumferentia et centrum est Deus qui est undique et nullibi.
2. Lucas, J.R. [1973]: *A Treatise on Time and Space*, Methuen.
3. Davies, P. [1985]: *About Time. Einstein's Unfinished Revolution*, Penguin.
4. Malament, D. [1977]: 'Causal Theories of Time ...', ***Noûs 11***, p.293.
5. Mercier, Treder, Yourgrau [1979]: *On General Relativity*, Akademie Verlag Berlin.
(my mentor, André Mercier: co-founder of CERN, founder of *Gen.Rel.Grav.*, initiator of the 50 yr. Jubilee of Spec.Rel., former secretary gen. of FISP (Fed.Int.Soc.Philos.).)
6. Mercier, A. [2000]: 'The Reconstruction of Space-Time as Time-Space', in:
Duffy & Wegener: *Recent Advances in Relativity Theory vol.I*, Hadronic Press, Fl. US.
7. Merleau-Ponty, J. (not M.) [1965]: *Cosmologie du XXme siecle*, Gallimard.
8. Milne, E.A. [1948/1951]: *Kinematic Relativity*, Oxford University Press.
9. Mohr, Georg [1672]: *Euclides Danicus,* Amsterdam (german transl. Copenhagen 1928) - anticipated the insights of L. Mascheroni [1797]: *Geometria del compasso*, by 125 years!
10. Øhrstrøm, P. [2000]: 'Tense Logic and Special Relativity', in:
Duffy & Wegener: *Recent Advances in Relativity Theory vol.I*, Hadronic Press, Fl. US.
11. Phipps, T.E. jr. [1986]: *Heretical Verities*, classic non-fiction libr., Urbana, US.
12. Rowlands, P. [2007]: *Zero to Infinity. The Foundations of Physics*, World Scientific.
13. Schutz, J.W. [1973]: *Foundations of Special Relativity*, Springer (Lect.Notes Math.361).
14. Selleri, F. [2009]: *Weak Relativity* (forthcoming - Apeiron?)
15. Törnebohm, H. [1963]: *Concepts and Principles ...*, Gothenburg Studies in Philosophy 2.
16. Törnebohm, H. [2000]: 'Infra-theories to the Special Theory of Relativity', in:
Duffy & Wegener: *Recent Advances in Relativity Theory vol.I*, Hadronic Press, Fl. US.
17. Ungar, A.A. [2009]: *Analytic Hyperbolic Geometry & Alb. Einstein's **SR***, World Scientif.
18. Walker, A.G. [1959]: 'Axioms for Cosmology', pp.308-321 in:
Henkin, Suppes & Tarski: *The Axiomatic Method*, North Holland.
19. Wegener, M.T. [1993]: 'Time and Harmony in Leibniz', in G. Seel & al., eds.:
L'Art. la Science et la Metaphysique (Festschrift to André Mercier), Peter Lang.
20. Wegener, M.T. [2004]: 'The Idea of a Cosmic Time', ***Found.Phys.34***,11,1777-99.
21. Wegener, M.T. [2007]: 'Big Bang versus Steady State', PIRT-Proc., Budapest.
22. Whitrow, G.J. [1961]: *The Natural Philosophy of Time*, Nelson/ Harper/ Oxford Univ.Pr..
23. Whitrow, G.J. [1972]: *What is Time?* Thames & Hudson/ Penguin.
24. Whittaker, E.T. [19]: *A History of the Theories of Aether & Electricity II*, Harper.
25. Winnie, J. A. [1970]: '***SR*** without One-Way Assumptions', in: ***Phil.Sc.37***, pp.81&223.

= // =